

# **BLACK SEA JOURNAL OF ENGINEERING AND TRANSPORT**

Science · Technology · Economics · Education

## **Volume 1, 2025**

Editor-in-Chief  
**Liliya Staneva, PhD**

**2025**  
**Burgas, Bulgaria**  
**Publisher: Black Sea Experts**

**ISSN 3033-2885**

# **BLACK SEA JOURNAL OF ENGINEERING AND TRANSPORT**

Science • Technology • Economics • Education

Editor-in-Chief:  
Liliya Staneva, PhD

Publisher:  
Black Sea Experts

Place of Publication:  
Burgas, Bulgaria

Web site:  
[www.bse-bg.com](http://www.bse-bg.com)

Email:  
[journal@bse-bg.com](mailto:journal@bse-bg.com)

ISSN 3033-2885

**Editor-in-Chief:**

Assoc. Prof. Dr. Eng. Liliya Staneva

**Editorial Board Members:**

Prof. Dr. Eng. Tsvetoslav Tsankov

Prof. Dr. Eng. Emil Marinov

Prof. Dr. Eng. Bogana Tabakova

Assoc. Prof. Dr. Ivelina Vardeva

Assoc. Prof. Dr. Eng. Yana Koleva

Assoc.Prof. Dr. Milena Dimitrova

Dr. Eng. Yuliyen Petrov

Dr. Eng. Ivan H. Ivanov

## Contents

|                                            |                                                                                                           |    |
|--------------------------------------------|-----------------------------------------------------------------------------------------------------------|----|
| <i>Liliya Staneva,<br/>Yuliyana Petrov</i> | Generalized Nets and Their Application in the Management of a Recording Studio                            | 5  |
| <i>Ivelina Vardeva</i>                     | Oracle Cloud Infrastructure: Intelligent Architecture for Efficiency, Security, and Autonomy in the Cloud | 8  |
| <i>Daniel Denev</i>                        | Adaptive Traffic Management Algorithms in Ultra-Reliable Low-Latency Networks                             | 12 |
| <i>Ivan Ivanov</i>                         | Corrosion on cylinder heads as an indicator of internal combustion engine reliability                     | 17 |
| <i>Stanko Pavlov,<br/>Liliya Staneva</i>   | Geometric Methods in Vector Calculus: Scalar and Vector Triple Products and Their Applications            | 21 |

# Generalized Nets and Their Application in the Management of a Recording Studio

Liliya STANEVA<sup>1</sup>, Yuliyana PETROV<sup>2\*</sup>

<sup>1</sup>Black Sea Experts, Burgas Bulgaria

<sup>2</sup>Black Sea Experts, Burgas, Bulgaria

Received: 10.10.2025

Revised: 18.10.2025

Accepted: 20.10.2025

Published: 25.10.2025



**Copyright:** © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

**Abstract:** Generalized nets, introduced by Academician Krassimir Atanasov [1], represent an established methodology for modeling and analyzing complex systems. This article explores their potential application in the management of recording studios, focusing on the optimization of recording, processing, and production workflows. The study highlights how generalized nets can enhance coordination between different operational units within a studio, ensuring efficient time and resource management. Recording studios operate as dynamic systems that require seamless integration of diverse technological and organizational components. Decision-making in such an environment must account for scheduling constraints, resource allocation, and task prioritization. By applying the principles of generalized nets, a structured approach to managing these processes can be established, including modeling key workflows, predicting potential challenges, and improving the distribution of human and technical resources.

**Keywords:** Generalized Nets, Recording Studio, Optimization, Process Management, Petri Nets Theory

5

## 1. INTRODUCTION

The article examines several key applications of Generalized Nets [1], [2], [3] in the environment of recording studios. These include:

- Modeling the workflow – assisting in structuring the production phases such as recording, mixing, mastering, and distribution;
- Optimization of scheduling – synchronizing recording sessions with the availability of performers;
- Automation of technical processes – managing sound effects and audio filters efficiently;
- Resource monitoring – ensuring the optimal utilization of studio spaces, equipment, and software.

By simulating this net, studio managers can anticipate inefficiencies in the workflow and implement corrective measures. Additionally, the study presents a case in which a recording studio works with multiple artists simultaneously, demonstrating how Generalized Net [6], [7] can be used to visualize and optimize session allocation. The analysis indicates that applying these models leads to improved operational efficiency and higher production quality.

A generalized model for optimizing workflows in a recording studio is proposed, aiming to support the analysis and planning of production stages. The developed net model represents a structured scheme of the workflow, designed to assist in optimizing production processes. In practical applications, specific management details may vary significantly depending on the studio, team, and type of projects. [11], [12]

The purpose of the article is not to prescribe a single working model for all studios but rather to provide a tool that can identify weaknesses, assist in resource planning, and enable experimentation with simulations before making actual changes.

## 2. MATERIALS AND METHODS

A generalized net model of the processes in a recording studio has been developed, introducing a set of transitions  $A = \{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$ , which describe the following processes:

$Z_1$  – “Checking for available recording studio”

$Z_2$  – “Recording in progress”

$Z_3$  – “Recording completed”

\*Corresponding author.

E-mail address: lanestieva@gmail.com

$Z_4$  – “Mixing in progress”

$Z_5$  – “Mastering in progress”

$Z_6$  – “Final product”

Each transition can have  $m$  input and  $n$  output positions, where  $m \geq 1$  and  $n \geq 1$ . The input positions for transition  $Z$  are denoted as  $l'_1, \dots, l'_i, \dots, l'_m$ , while the output positions are labeled as  $l''_1, \dots, l''_j, \dots, l''_n$ . An input position that does not receive an arc is considered an entry point of the net, while an output position with no outgoing arc is considered an exit. In the network positions, tokens can be present and move from input to output positions of the transitions. In Generalized Nets, tokens represent information units that pass through the system transitions and carry characteristic data about the process state. When the input positions contain a sufficient number of tokens and the specified transition time occurs, the tokens from the input positions become eligible to move to the output positions. This movement of tokens from input to output positions is called transition activation.

Initially, the tokens entering the net through the input positions have initial characteristics. With each transition within the net, they acquire new characteristics, making each token unique with its own history.

The net model has been created using GNDraw, a software tool designed for the graphical modeling of Generalized Nets Fig. 1. The transitions are described as follows:

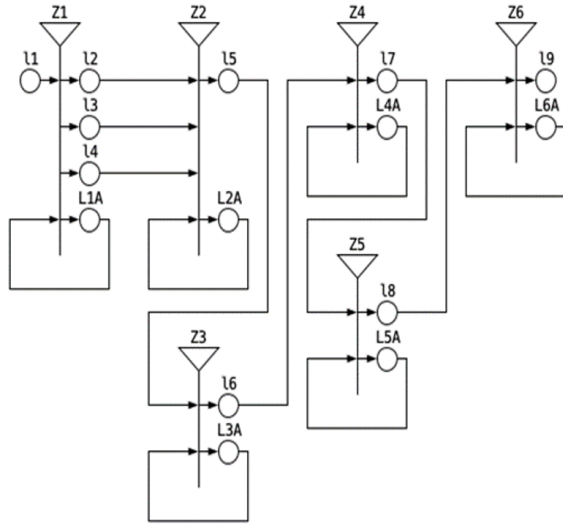
$$Z_1 = \langle \{l_1, L_{1A}\}, \{l_2, l_3, l_4, L_{1A}\}, R_1, M_1 \wedge (l_1) \vee (L_{1A}) \rangle$$

$$R_1 = \begin{array}{c|cccc} & l_2 & l_3 & l_4 & L_{1A} \\ \hline l_1 & false & false & false & true \\ L_{1A} & W_{1A,2} & W_{1A,3} & W_{1A,4} & true \end{array}$$

$W_{1A,2}$  – “A free recording studio is available”.

$W_{1A,3}, W_{1A,4}$  – “The necessary conditions for creating a sound recording are available”.

The prerequisite is the availability of a free studio space. After the transition is activated, the core moves to the “Recording in Process” position, with a current characteristic corresponding to the type of recording – vocals, string instruments, or drums. Once this condition is met, the incoming core from position  $l_1$  enters position  $L_{1A}$ .



**Fig. 1.** Graphical representation of a Generalized Net for managing processes in a recording studio, created using GNDraw.

After the transition, the cores  $l_2, l_3, l_4$  exiting from position  $L_{1A}$  are assigned the current characteristics “vocal recording”, “string instrument recording”, and “drum and keyboard recording”.

$$Z_2 = \langle \{l_2, l_3, l_4, L_{2A}\}, \{l_5, L_{2A}\}, R_2, M_2 \wedge (l_2, l_3, l_4) \vee (L_{2A}) \rangle$$

|         | $l_5$    | $L_{2A}$        |
|---------|----------|-----------------|
| $R_2 =$ | $l_2$    | false           |
|         | $l_3$    | false           |
|         | $l_4$    | false           |
|         | $L_{2A}$ | $W_{2A,5}$ true |

$W_{2A,5}$  – “The cores  $l_2, l_3, l_4$  are available for the start of the recording”.

The incoming cores from positions  $l_2, l_3, l_4$  enter  $L_{2A}$ . After the transition, the cores merge into a single core  $l_5$  with the current characteristic “the recordings are ready.”

$$Z_3 = \langle \{l_5, L_{3A}\}, \{l_6, L_{3A}\}, R_3, M_3 \wedge (l_5) \vee (L_{3A}) \rangle$$

|         | $l_5$    | $L_{3A}$        |
|---------|----------|-----------------|
| $R_3 =$ | $l_6$    | false           |
|         | $L_{3A}$ | $W_{3A,6}$ true |

$W_{3A,6}$  – “The recordings are complete”.

The incoming core  $l_5$  enters position  $L_{3A}$ . After the transition, the core exiting to position  $l_6$  takes the current characteristic “records entering for mixing.”

$$Z_4 = \langle \{l_6, L_{4A}\}, \{l_7, L_{4A}\}, R_4, M_4 \wedge (l_6) \vee (L_{4A}) \rangle$$

|         | $l_7$    | $L_{4A}$        |
|---------|----------|-----------------|
| $R_4 =$ | $l_6$    | false           |
|         | $L_{4A}$ | $W_{4A,7}$ true |

$W_{4A,7}$  – “Processes individual sound recordings (vocals, drums, guitars)”.

The incoming core  $l_6$  enters position  $L_{4A}$ . After activating the transition, the exiting core  $l_7$  assumes the current characteristic “mixed recordings”.

$$Z_5 = \langle \{l_7, L_{5A}\}, \{l_8, L_{5A}\}, R_5, M_5 \wedge (l_7) \vee (L_{5A}) \rangle$$

|         | $l_8$    | $L_{5A}$        |
|---------|----------|-----------------|
| $R_5 =$ | $l_7$    | false           |
|         | $L_{5A}$ | $W_{5A,8}$ true |

$W_{5A,8}$  – “focuses on the overall sound balance”.

The incoming core  $l_7$  enters position  $L_{5A}$ . After activating the transition, the exiting core  $l_8$  assumes the current characteristic “the mix is optimized and balanced.”

$$Z_6 = \langle \{l_8, L_{6A}\}, \{l_9, L_{6A}\}, R_6, M_6 \wedge (l_9) \vee (L_{6A}) \rangle$$

|         | $l_9$    | $L_{6A}$        |
|---------|----------|-----------------|
| $R_6 =$ | $l_8$    | false           |
|         | $L_{6A}$ | $W_{6A,9}$ true |

$W_{6A,9}$  – “the sound recording is ready for release”.

The incoming core  $l_8$  enters position  $L_{6A}$ . After the transition is activated, the exiting core  $l_9$  assumes the current characteristic “the mix processing is complete.”

The possibilities for integrating artificial intelligence (AI) [4], [5] in the management of recording studios through neural net represent a significant advancement in process automation. AI can be used for:

- Automated resource allocation – intelligent algorithms can analyze the studio’s workload and dynamically propose optimal schedules for recording, mixing, and mastering.
- Sound and quality analysis – AI-based tools can monitor audio quality in real time and automatically suggest corrections to the mix or mastering levels.
- Prediction of delays and optimization – through machine learning, the system can forecast potential issues in the workflow and propose preventive measures.
- Voice and text commands – AI assistants can facilitate the work of sound engineers by executing commands to change mixer settings or activate specific effects.

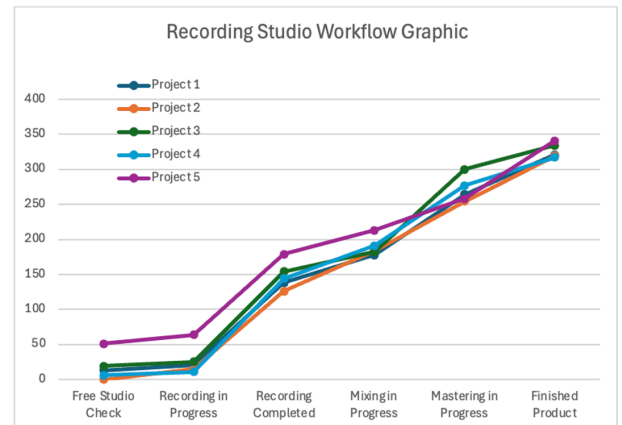
The integration of artificial intelligence in the management of recording studios will lead to higher efficiency, reduced errors, and optimized time expenditures. In the future, these technologies will enable the creation of fully autonomous studio environments, where AI will manage the entire production process with minimal human intervention.

### 3. RESULTS AND DISCUSSION

The workflow in a recording studio is simulated using the stages described in a generalized net [8], [13],[14]. Time data necessary for each process involved in creating a musical product (song) is provided (Table 1).

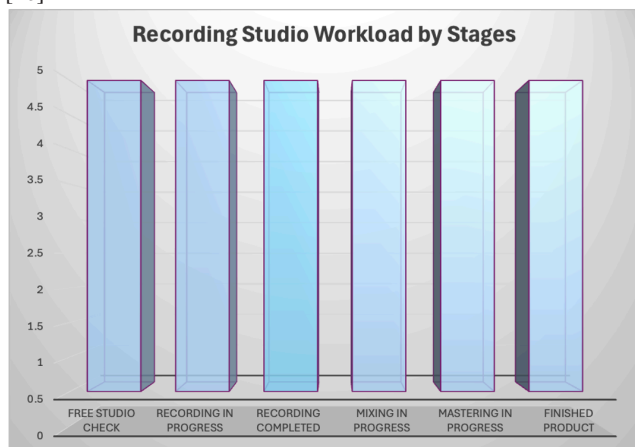
**Table 1.** Workflow in a recording studio

| Project               | Project 1 | Project 2 | Project 3 | Project 4 | Project 5 |
|-----------------------|-----------|-----------|-----------|-----------|-----------|
| Free Studio Check     | 13        | 0         | 19        | 6         | 51        |
| Recording in Progress | 20        | 15        | 25        | 11        | 64        |
| Recording Completed   | 138       | 126       | 154       | 144       | 179       |
| Mixing in Progress    | 178       | 184       | 182       | 191       | 213       |
| Mastering in Progress | 264       | 254       | 300       | 277       | 258       |
| Finished Product      | 321       | 319       | 334       | 317       | 341       |



**Fig. 2.** Analysis of the Studio’s Workload by Process Stages.

The first chart (Fig. 2) shows the time distribution of projects by stage. It visualizes how each project moves through the different stages over time, allowing the identification of potential "bottlenecks" where multiple projects overlap in the same process. [9], [10]



**Fig. 3.** Time Distribution of Projects by Stages in the Recording Studio.

The second chart (Fig. 3) analyzes the studio's workload by stage, showing how many projects are active in each stage. If a stage (for example, mixing) is overloaded, it may indicate that additional resources or a better time allocation is needed. To address this issue, it is necessary to optimize the schedules so that mixing and mastering do not take as much time. To reduce this load, more engineers or studio resources can be added to distribute the workload. When mixing and mastering take too much time, parallel processes or automated AI solutions can be introduced; in the future, artificial intelligence might be used to analyze the workload in real time and make automatic adjustments. This will allow faster processing of recording projects without any loss in quality. In the next phase of the research on this problem, an algorithm for an automated recording studio schedule may be proposed. [15], [16]

#### 4. CONCLUSIONS

Generalized nets can provide a useful tool for analyzing and optimizing standard production processes in a recording studio. However, managing a studio in real conditions is often considerably more flexible and dynamic, involving human factors, unforeseen changes, and the parallel execution of tasks. Therefore, the model should be viewed as an optimization tool. In the future, these models could be integrated with artificial intelligence systems for more automated management and workload prediction in a studio. The publication applies the use of generalized nets to manage and optimize processes in a recording studio. Most research in this area has focused on industrial and technical systems, but here they are applied in a creative and technological environment.

#### Acknowledgments

In this section, we would like to thank all of our loved ones for their support and motivation throughout the process of our scientific research.

#### REFERENCES

[1] **Atanassov K.** *Generalized nets.* World scientific, 1991

- [2] **Reiss J.**, Intelligent Systems for Mixing Audio, *J. Audio Eng. Soc.*, vol. 59, no. 10, pp. 744–749, 2011.
- [3] **Hristova G.** Generalized Net Model of Editing Process in a Publishing House *Scientific Atlas*, no. 9, 2024. ISSN: 2738-7518.
- [4] **Nimr H.**, Defuzzification of the outputs of fuzzy controllers, *presented at the 5th Int. Conf. on Fuzzy Systems, Cairo, Egypt, 1996.*
- [5] **Murata T.**, "Petri nets: Properties, analysis and applications," *Proc. IEEE*, vol. 77, no. 4, pp. 541–580, Apr. 1989.
- [6] **Jensen K.**, *Coloured Petri Nets: Basic Concepts, Analysis Methods and Practical Use.* Berlin: Springer, 1997.
- [7] **Slavova V, Atanasov K.**, "A generalized net for modeling working memory and language processing," in *Proc. 7th Int. Conf. Cognitive Modeling in Linguistics*, vol. 9, pp. 90–101, 2004.
- [8] **Atanassov A, Gluhchev G, Hadjitodorov S, Shannon A, Vasilev V.**, "Generalized nets in image processing and pattern recognition," in *Proc. 6th Int. Workshop on Generalized Nets*, Sofia, vol. 17, pp. 47–60, Dec. 2005.
- [9] **Denev D., Tsankov Ts.**, "A Survey of Data Quality in Digital Processing of Acoustic Signals," in *2022 Int. Conf. on Communications, Information, Electronic and Energy Systems (CIEES)*, Veliko Tarnovo, Bulgaria. IEEE, 2022. ISBN: 978-1-6654-9149-5. [Online]. Available: <https://doi.org/10.1109/CIEES55704.2022.9990685>
- [10] **Denev D., Docheva A.**, "Software Processing of Audio Signals in Digital Audio Workstation Studio One Through Effect Plugins," in *Proc. Conf. with Int. Participation H2024*, vol. 2, Communication and Computer Technologies, ISSN: 1314-3921.
- [11] **Denev D.**, "Music Scoring for Film Using Fruity Loops Studio," *Annual of Konstantin Preslavski Univ. of Shumen*, vol. XI E, 2021. ISSN: 1311-834X.
- [12] **Denev D., Tsankov Ts.**, "Synthesis and analysis of linear, discrete and time-invariant systems, used in the field of communications, using MATLAB and signal processing toolbox," *J. Scientific and Applied Research*, vol. 22, pp. 72–80, 2022. ISSN: 1314-6289.
- [13] **Atanassov K.**, "The Foreign Object Principle and its generalized net interpretation," in *Proc. 1st Int. Workshop on Generalized Nets*, Sofia, vol. 9, pp. 34–36, July 2000.
- [14] **Atanassov K., Aladjov H.**, "A generalized net for machine learning of the process of mathematical problems solving: on an example with a Smarandache problem," *Smarandache Notions Journal*, vol. 12, pp. 338–344, 2001.
- [15] **Fidanova S., Atanassov K., Kirilov L., Slavova V, Ivanov V.**, "Generalized Net Model of the Consequences of Earthquake," in *Int. Symp. on Bioinformatics and Biomedicine*, Cham: Springer Nature Switzerland, 2022, pp. 281–292.
- [16] **Lubich M., Papazov V., Popov E., Georgieva R., Dmitrenko D., Bojkov B. et al.**, "A Generalized Net Model of the Prostate Gland's Functioning," *Mathematics*, vol. 10, no. 3, p. 479, 2022.

# Oracle Cloud Infrastructure: Intelligent Architecture for Efficiency, Security, and Autonomy in the Cloud

Ivelina VARDEVA<sup>1</sup>

<sup>1</sup>CNSYS, Burgas Bulgaria

**Abstract:** This article examines Oracle Cloud Infrastructure (OCI) as a second-generation platform that combines architectural isolation, high performance, and economic efficiency. A comparative analysis is conducted between OCI and leading cloud solutions such as AWS, Microsoft Azure, and Google Cloud Platform, emphasizing structured architecture, the balance between costs and performance, as well as the resilience of solutions in complex corporate environments. Using mathematical models for efficiency and Total Cost of Ownership (TCO), the study demonstrates that OCI achieves significantly better results thanks to low-latency communication, transparent regional pricing, and automated infrastructure management. The introduction of modern functionalities in 2025—such as Lakehouse analytics and Zettascale HPC supercomputer clusters—expands OCI’s capabilities for data processing and high-density computing. The Dedicated Region feature allows for a complete cloud solution to be deployed locally in the client’s data center with full compliance in security and access. Real-world applications in areas such as finance, logistics, and scientific research confirm the platform’s reliable performance and economic advantages. In conclusion, OCI is established as a strategic foundation for the next decade, unifying high performance, security, and transparency in an environment that redefines the role of cloud technologies in the corporate sector.

**Keywords:** OCI, RDMA communication, Cloud architecture, Security in the Cloud, Autonomy in the Cloud

Received: 20.10.2025  
Revised: 23.10.2025  
Accepted: 25.10.2025  
Published: 27.10.2025



**Copyright:** © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

8

## 1. INTRODUCTION

Over the past decade, the evolution of cloud technologies has established a new paradigm in enterprise resource management that integrates automation, scalability, and security. Within this transformation, Oracle Cloud Infrastructure (OCI) has emerged as a second-generation architecture designed to combine high performance, low latency, and a high degree of autonomy in infrastructure management. This paper examines the advantages of OCI in comparison with competing platforms such as AWS, Microsoft Azure, and Google Cloud Platform, focusing on architectural efficiency, cost–performance balance, and the resilience of solutions in multi-layered enterprise environments. The analysis is based on a review of publicly available data and technical documentation, including performance metrics integrated with mathematical models such as Efficiency (E) and Total Cost of Ownership (TCO). The discussion also addresses new functionalities introduced in 2025, including Generative AI Services, Lakehouse Analytics, and the Dedicated Region capabilities for governmental and corporate domains. The findings indicate that OCI delivers superior performance under critical workloads and provides enhanced integration with artificial intelligence services, while simultaneously optimizing cost structures through unified regional pricing. The conclusion highlights Oracle Cloud Infrastructure as a strategic cloud environment for the next decade, capable of unifying security, intelligent management, and economic efficiency across modern organizational cloud ecosystems.[3]

## 2. OCI ARCHITECTURE AND CONCEPT

### Definition and Principles of Second-Generation Cloud

Oracle Integration Cloud Generation 3 (Gen3) establishes a cloud-native, container-based architecture powered by Kubernetes and Oracle Cloud Infrastructure (OCI). This enables dynamic autoscaling, service resiliency, and true microservice isolation, resulting in superior performance, faster deployments, and enhanced automation for integration processes. Gen3 advances the prin-

ciple of isolation by leveraging deeply separated, auto-managed microservices—each integration operates within its own container. This not only eliminates the “noisy neighbour” problem but enables dynamic, automated responsiveness to fluctuating traffic, building on the strict customer/management isolation model of Gen2.

### Key Architectural Components

**Compute Layer:** Gen3 deploys dynamic container orchestration for seamless scaling of virtualized and bare-metal resources or serverless environments, maximally optimizing workload efficiency under load. **Networking Layer:** The OCI Virtual Cloud Network is enhanced with zero-trust security, automated policy enforcement, and design for seamless multi-cloud connectivity, supporting complex enterprise network topologies. **Storage Layer:** Storage systems in Gen3 are optimized for large data payloads (e.g., REST/SOAP up to 100MB), high-speed transactional workloads, and include built-in lifecycle management for object-based and transactional storage.

**Autonomous Database and AI:** Integrated AI capabilities automate resource distribution, retry logic, diagnostics, and adaptive workflow optimization for both database and integration management. [5]

### Performance Mathematical Model

For Gen3, efficiency (E) improves over realized performance (R) and infrastructure cost (C), anchored by cloud-native networking, autoscaling, and automated resource management:

$$E = \frac{R}{C} \quad (1)$$

Gen3 delivers higher efficiency with:

- Independent, containerized infrastructure with minimal overhead;
- Automated resource/latency management even at peak scale;
- Clear, all-inclusive pricing metrics and no hidden usage fees.

### Architectural Advantages of Gen3

**Automated Security:** Microservices with isolated duties, au-

\*Corresponding author.

E-mail address: ivardeva@gmail.com

tomated access enforcement, and self-healing mechanisms.

**Predictable Performance:** Consistently low latency and scale achieved through per-integration autoscaling, even during spikes.

**Economic Transparency:** Unified pricing and clear KPIs for data integration and transfer, ensuring improved cost visibility and predictability.

**Multi-Cloud Interoperability:** Native OCI integration with Azure, Google Cloud, AWS, without the need for re-engineered APIs or migration of business logic.

**Expanded Automation and DevOps:** Direct integration with CI/CD pipelines (GitOps, webhooks), multi-region disaster recovery, and rapid deployment of new automations and integrations.

Gen3 sets a new benchmark for scalable, secure, and predictable integration—delivering enterprise-grade flexibility, rapid automation, and cloud-native optimization. This platform best serves modern organizations requiring fast adaptation, robust security, and traceable integration costs.[1]

### 3. ADVANTAGES OF ORACLE CLOUD INFRASTRUCTURE

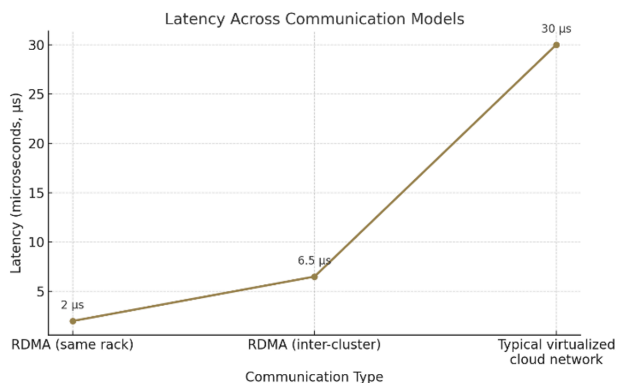
#### Performance and Low Latency via RDMA Networks

Oracle Cloud Infrastructure leverages Remote Direct Memory Access (RDMA) networks to facilitate direct data exchange between system memories, eliminating processor and operating system involvement. This configuration achieves minimal latency—below 2 microseconds within a single server rack and 6 to 7 microseconds between clusters—which is essential for financial computations, scientific analysis, and artificial intelligence models. OCI delivers performance comparable to on-premises infrastructure while maintaining the flexibility characteristic of cloud environments. Each customer receives a dedicated virtual network and an isolated control plane, which eliminates risks associated with unauthorized access and the “noisy neighbor” phenomenon. Principles such as least privilege and zero trust are applied, ensuring that access and operations are restricted to explicitly defined zones; this approach safeguards against both attacks and configuration errors.

Oracle Cloud Infrastructure enforces a consistent pricing policy across all regions, featuring fixed service tariffs and complimentary intra-regional data transfers. This clarity facilitates budgeting and distinguishes OCI from other providers. The Universal Credits model enables consumption-based payment for a variety of services.

Through Oracle Interconnect for Azure, direct connectivity is established between Oracle and Microsoft platforms, permitting seamless integration of applications and services without routing data over the public internet. This results in low latency and unified management and support systems. The outcome is a flexible and secure multicloud ecosystem without compromises on performance or security.

Line chart “Latency by Communication Type” with the X-axis representing communication type and the Y-axis showing latency (in microseconds), Fig1.



**Fig. 1.** Latency for different types of communication in microseconds.

The chart includes three data points:

RDMA (single rack): 2  $\mu s$

RDMA (inter-cluster): 6.5  $\mu s$

Typical virtualized cloud network: 30  $\mu s$

Oracle Cloud Infrastructure combines high-speed networks, robust isolation, transparent pricing, and Azure integration, making it a practical alternative for organizations seeking control, speed, and predictability.

### 4. INNOVATIVE FEATURES AND DEVELOPMENTS IN 2025

#### Generative AI Services and Oracle AI Lakehouse

In 2025, Oracle is consolidated as a strategic infrastructure partner for enterprise artificial intelligence with the launch of Oracle AI Database 26ai and Autonomous AI Lakehouse. The new database integrates generative AI natively within Oracle Database, enabling automatic generation of SQL queries, test data, and analytic summaries without externalizing sensitive information. AI Lakehouse unifies structured and unstructured data via Apache Iceberg, offering a comprehensive environment for analysis and model training.

Oracle Cloud Infrastructure introduces Zettascale HPC Superclusters equipped with NVIDIA Blackwell, H200, and AMD MI300X GPU accelerators, reaching up to 260 ExaFLOPS and 50Pb/s throughput through NVLink. This

RDMA and RoCE-based architecture delivers linear scaling for large language models and scientific simulations, granting access to high performance computing (HPC) without the need for proprietary supercomputer resources.

OCI Dedicated Region 25 enables the deployment of a complete cloud infrastructure on-premises at the client’s datacenter with a minimal footprint (three RAC/Real Application Clusters modules), providing access to the full suite of Oracle public services. This solution offers comprehensive control over data and is tailored for sectors with rigorous regulatory requirements, such as banking, governmental, and healthcare institutions. It supports the same APIs and SLAs as the public OCI Cloud.

Compared to competitors like AWS, Azure, and GCP, Oracle provides built-in AI integration within the database (26ai, Select AI Agent), Lakehouse capabilities based on Apache Iceberg, HPC scaling up to 260 ExaFLOPS, and complete data sovereignty via Dedicated Region. Competing platforms typically deliver partial or limited offerings in these areas.

Oracle Cloud Infrastructure transitions from a standard “cloud as a service” approach to the concept of “data and intelligence as a service,” integrating security, performance, and artificial intelligence within a unified ecosystem. This positions Oracle among the most innovative cloud platforms when compared to AWS, Azure, and GCP, Table 1.

Oracle Cloud Infrastructure’s innovations in 2025 define a strategic shift: from pure infrastructure to an integrated intelligent cloud. The combination of in-database AI, HPC-scale resources, and autonomous solutions positions Oracle as a technological integrator, uniting security, compute, and artificial intelligence within a unified ecosystem that rivals the largest public clouds.

The trend is clear—Oracle is reshaping the market by shifting the focus from “cloud as a service” to “data and intelligence as a service.”[4]

### 5. MATHEMATICAL AND ECONOMIC ASSESSMENT

The economic evaluation of Oracle Cloud Infrastructure (OCI) is based on an extended Total Cost of Ownership (TCO) model that incorporates all direct and indirect costs associated with the deployment and maintenance of the infrastructure. This model is designed to measure the actual value of OCI compared to traditional on-premises solutions and competing cloud platforms.

$$TCO = (C_I + C_O + C_S) - (S_E + C_T) \quad (2)$$

where:

- $C_I$  – infrastructure costs (compute resources, storage, network services);
- $C_O$  – operational expenses (administration, maintenance);
- $C_S$  – costs associated with security, licensing, and access control;

Table 1. Comparison across major cloud platforms

| Indicator              | OCI (2025)                                                  | AWS                         | Azure                            | GCP                      |
|------------------------|-------------------------------------------------------------|-----------------------------|----------------------------------|--------------------------|
| AI Integration         | Built into the database (AI Database 26ai, Select AI Agent) | Via SageMaker and Bedrock   | Copilot and Azure OpenAI Service | Vertex AI                |
| Lakehouse Architecture | Autonomous AI Lakehouse with Apache Iceberg format          | AWS Lake Formation          | Microsoft Fabric                 | BigLake                  |
| HPC GPU Scale          | Up to 260 ExaFLOPS, 131k GPUs                               | ~50 ExaFLOPS                | ~35 ExaFLOPS                     | ~40 ExaFLOPS             |
| Dedicated Region       | Minimum 3 racks, full OCI functionality                     | Outposts (limited services) | Azure Stack Hub                  | Anthos / Edge nodes      |
| Sovereign Cloud        | Complete local data isolation                               | Partial                     | Under EU agreements              | Limited to hybrid models |

- $S_E$  – efficiency gains (higher performance with lower resource consumption);
- $C_T$  – cost savings in time due to automation and reduced manual management.

According to conducted assessments, migration from on-premises infrastructure to OCI typically results in a 30–40% reduction in TCO over a four-year period, primarily due to lower network service fees and automated database management. Oracle Cloud offers tools such as the OCI TCO Calculator and an integrated FinOps console, enabling businesses to simulate various scenarios and forecast actual consumption.

**Efficiency Analysis Relative to Alternative Platforms**

Cost efficiency may be expressed using the ratio from (1):  $R$  denotes productive output (number of transactions, computations, IOPS), and  $C$  is the total direct cost. Comparative infrastructure evaluations reveal that Oracle delivers 50% lower compute costs, 70% lower block storage costs, and up to 80% lower network traffic costs than competitors AWS, Azure, and Google Cloud. Additionally, 10 TB of monthly outbound traffic is provided free of charge, and standardized pricing across regions eliminates hidden fees that often increase overall costs with other providers.

Consequently, efficiency ( $E$ ) for identical workloads is approximately 1.5 to 2 times higher with OCI compared to AWS or Azure, especially in scenarios involving large datasets and databases with high I/O intensity.

**Graphical Visualization of “Cost-Performance” Ratio**

The line chart in Fig. 2 illustrates the comparative relationship between the price per unit performance – COST (USD / 1M IOPS) and actual performance measured in transactions per second – Performance (TPS) for four major cloud platforms: OCI, AWS, Azure, and GCP.[6]

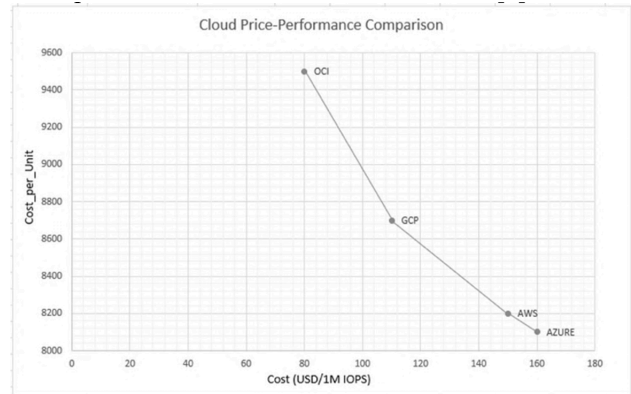


Fig. 2. Cost-performance ratio across major cloud platforms

The chart in Fig. 2 clearly demonstrates that OCI offers the best efficiency at a significantly lower cost, occupying the “high performance – low cost” quadrant. AWS and Azure achieve comparable levels of performance but at much higher expenses. GCP holds an intermediate position, providing balanced yet more variable efficiency compared to the other providers.

This evaluation goes beyond simple price comparisons; it serves to determine the long-term economic benefit of investing in OCI. The application of the TCO model enables:

- objective budgeting for a 3–5 years period;
- identification of hidden operational savings (such as through automation and reduced downtime);
- financially substantiated planning for multicloud or hybrid strategies.

Oracle positions its cloud efficiency model as one that unites technical and economic factors—not through promises of a “cheaper cloud,” but via demonstrable metrics for real returns, measured by TCO and the effectiveness ratio (1). In this way, the platform becomes a predictable optimization tool, where the business evaluates not just price, but the value gained for every unit of resource used. [5]

**6. APPLICATIONS AND REAL-WORLD USE CASES**

In 2025, more than six thousand companies worldwide rely on Oracle Cloud Infrastructure for key processes—from banking and logistics to energy and research centers. Within the banking sector, OCI delivers robust infrastructure for payment systems and real-time financial transaction analysis. Banks in Europe and Asia utilize RDMA networks for instant payments and distributed ledger services. In logistics, firms such as PRIO Energy implement OCI to manage ERP and communications across commercial sites, achieving comprehensive real-time traceability of flows. In scientific domains, OCI HPC infrastructure supports simulations in bioinformatics and climate modeling, where uninterrupted processing of large datasets is essential.

**Autonomous Databases and Maintenance Automation**

Oracle Autonomous AI Database automates all core maintenance activities, including backup, updates, tuning, and security. This self-managing architecture significantly reduces human intervention and operational costs associated with administration. Practically, in enterprise environments such as Star CRM or large SaaS vendors, databases self-optimize based on workload, eliminating the need for manual management. As a result, companies report an average reduction of 30–40% in maintenance expenses and meaningful stabilization of performance.

**Practical Scenario: Oracle Database@Azure**

Oracle Database@Azure enables the deployment of a complete Oracle Database environment on Microsoft Azure infrastructure. This platform allows organizations to migrate existing Oracle databases to Azure without modifying architecture or functionality. The solution features low latency and direct connectivity between both ecosystems. Applications in Azure have access to Exadata databases in OCI via private network, while management remains unified. This supports hybrid and multicloud strategies—for example, combining Azure analytics services with core business data on OCI.

The model is actively employed by international enterprises for use cases in commerce, pharmaceuticals, and telecommunications,

where cloud integration lowers costs and simplifies data management. The applications of Oracle Cloud Infrastructure are not theoretical: they yield tangible, measurable benefits—enhanced efficiency, automated maintenance, and seamless integration with other environments.

Thanks to its architecture, OCI demonstrates practical value both in mission-critical systems and hybrid solutions, where reliability and low latency are decisive success factors.[2]

## 7. ANALYSIS AND FUTURE RESEARCH DIRECTIONS

### Optimization of AI Resources Through Mathematical Models

The evolution of computing architectures within Oracle Cloud Infrastructure opens opportunities for mathematical analysis of AI resource efficiency. The application of regression models and stochastic optimization can aid in forecasting the most suitable distribution of compute power across tasks—particularly in HPC and large language model training.

This research is based on the following function:

$$F(x) = \alpha_1 C_{\text{GPU}} + \alpha_2 T_{\text{latency}} + \alpha_3 E_{\text{power}} \quad (3)$$

where variables represent cost, latency, and energy consumption. This approach enables determination of the optimal balance between expense and speed. Such a model would allow for dynamic management of GPU clusters and more accurate prediction of real-world AI workload consumption—areas where Oracle has already implemented Acceleron and RoCEv2-fabric approaches.

### Potential for Integration in Hybrid and Multicloud Environments

Oracle is entering a new phase, where multicloud strategies are becoming the norm. With the advent of Multicloud Universal Credits, organizations can leverage Oracle Database, AI Lakehouse, and infrastructure services across AWS, Azure, and Google Cloud without separate licences or reconfiguration.

Future studies could assess the economic and technical viability of this model—for instance, evaluating how unified licensing affects TCO in distributed computational environments. Of particular interest is benchmarking cloud latency across vendors and seeking optimal communication models where multicloud integration does not increase costs.

### Measuring Efficiency with Regression and Probabilistic Methods

Computational workloads in OCI can be analyzed through a combined regression approach that tracks dependencies between cost, latency, and scalability. For example:

$$R = \beta_1 P + \beta_2 S + \beta_3 L + \varepsilon \quad (4)$$

where:

- $R$  is the effective outcome;
- $P$  is computational power;

- $S$  is storage consumption;
- $L$  is latency;
- $\varepsilon$  represents deviation, or unpredictability in cloud network behaviour.

Through probabilistic modeling, these relationships can be applied to different scenarios—AI workloads, databases, or edge computing. Such analyses have practical applications: they help select the optimal configuration and identify threshold conditions under which Oracle Cloud Infrastructure maintains an economically efficient advantage over competitors.

Advancements in Oracle's cloud infrastructure provide ample ground for rigorous research—from mathematical resource modeling to evaluating multicloud integration and forecasting efficiency. The results of such investigations would support the optimization of future AI environments and reinforce Oracle's role as a technological foundation for analytics and computational innovation in the coming decade.

## 8. CONCLUSIONS

The study presents a comprehensive perspective on the development of Oracle Cloud Infrastructure as a technological platform combining high-performance computing, scalable infrastructure, and data protection. A key contribution is the definition of a model that illustrates the transition from the traditional cloud services concept to an integrated approach for managing data and compute resources.

The analysis delineates Oracle's competitive advantages over major market providers in metrics such as performance, latency, and total cost of ownership. It highlights the significance of architectural choices like RDMA networks, Dedicated Region, and the Lakehouse platform, which enable high efficiency and security in corporate data processing.

Practical outcomes demonstrate the applicability of the infrastructure across sectors—including public administration, financial services, and scientific computing—as well as its potential for integration into multicloud environments. Thus, the study substantiates the role of Oracle Cloud Infrastructure as a strategic foundation for building resilient, predictable, and scalable enterprise systems.

## REFERENCES

- [1] Cloud Economics, Oracle, 2025
- [2] <https://azure.microsoft.com/en-us/solutions/oracle/>
- [3] <https://www.oracle.com/cloud/why-oci/>
- [4] IT Executive's Guide to Oracle Cloud Infrastructure, Oracle, 2025
- [5] **Patt D., Szurley M.**, The Business Value of Oracle Autonomous Database, Business Value Executive Summary, 2025
- [6] **Sravana Y.**, Oracle OIC Gen2 to Gen3 Upgrade: Strategies and Best Practices, International Journal of Science and Research Archive, 2025, 16(01), 2051-2059, <https://doi.org/10.30574/ijrsra.2025.16.1.2200>

# Adaptive Traffic Management Algorithms in Ultra-Reliable Low-Latency Networks

Daniel DENEV<sup>1</sup>

<sup>1</sup>Konstantin Preslavski University of Shumen, Shumen, Bulgaria

**Abstract:** Ultra-reliable low-latency communications (URLLC) are a key component in the evolution of 5G and future 6G networks, providing critical connectivity for industrial automation, autonomous vehicles, and telemedicine. A key challenge in these networks is the efficient management of traffic in the presence of dynamic network conditions, limited resources, and stringent latency and reliability requirements. This paper reviews current approaches to adaptive traffic management based on optimization algorithms, machine learning, and network virtualization. An analytical framework is proposed to evaluate algorithmic performance, taking into account the interplay between quality of service (QoS), quality of experience (QoE), and network structural resilience.

**Keywords:** Adaptive management, Traffic forecasting, Optimization, Machine learning, Reliability

## 1. INTRODUCTION

The rapid evolution of mobile communications over the past decade has led to the deployment of fifth-generation (5G) networks and has outlined the conceptual foundations for future 6G architectures. One of the most critical components of these technologies is Ultra-Reliable Low-Latency Communications (URLLC), designed to meet extremely stringent requirements in terms of latency, reliability, and service continuity. These requirements are vital for a wide range of industrial and societal applications – including autonomous transportation, intelligent manufacturing systems, remote surgery, and smart energy management. Despite significant advances in radio access technologies and network architectures, efficient traffic management remains a fundamental challenge within URLLC environments. Dynamic and often unpredictable network conditions can lead to delay variations, congestion, and packet loss. [2], [6]

Traditional routing and load control mechanisms, typically based on static or semi-static policies, prove inadequate in scenarios where reaction times must remain within milliseconds and reliability must exceed 99.999%. This has necessitated the development of novel, flexible, and intelligent algorithms for adaptive traffic management. Adaptive algorithms have the potential to redefine Quality of Service (QoS) and Quality of Experience (QoE) assurance in next-generation networks. Through dynamic resource reallocation, traffic load prediction, and real-time optimization of network parameters, such algorithms enable latency minimization and enhanced system resilience. Furthermore, the integration of machine learning techniques and network virtualization expands the capabilities of self-organizing systems that can autonomously adapt to rapidly changing conditions without human intervention.

However, the design and deployment of such approaches pose significant scientific and engineering challenges. These include balancing algorithmic complexity with available computational resources, ensuring hardware acceleration for real-time processing, and achieving protocol-level standardization for evaluation and implementation. This paper examines the key research directions in the development of adaptive traffic management algorithms for URLLC networks, presents an analytical framework for their evaluation, and discusses the prospects for their application in critical industrial and societal systems. [3], [5]

## 2. THEORETICAL FOUNDATIONS OF ULTRA-RELIABLE LOW-LATENCY COMMUNICATIONS

Ultra-Reliable Low-Latency Communications (URLLC) are defined as one of the three main pillars of the fifth generation of mo-

bile networks (5G), along with massive Machine-Type Communications (mMTC) and enhanced Mobile Broadband (eMBB). The main goal of URLLC is to provide communications with extremely low latency (below 1 millisecond) and reliability reaching or exceeding 99.999%. This makes them fundamental for mission-critical applications, where even minimal delay or data loss could lead to severe consequences – for example, in telemedicine, autonomous transportation systems, and industrial process control. Unlike eMBB, which focuses on high throughput, and mMTC, which prioritizes massive device connectivity with low speed and latency requirements, URLLC is characterized by stringent end-to-end communication requirements. The key parameters include:

- End-to-end latency, which must be minimized;
- Packet Error Rate (PER) – the probability of transmission error, which must be reduced to the order of  $10^{-5}$  or lower;
- Availability and service continuity, which are critical for the reliability of real-time systems.

The mathematical foundations of URLLC are based on classical information theory and wireless channel modeling. While Shannon's theorem defines the limits of channel capacity under infinite codeword lengths, URLLC requires the use of concepts from finite blocklength theory. These models show that for small packets—typical in URLLC applications—the trade-off between latency, reliability, and capacity becomes stricter, demanding new coding and modulation techniques. In addition, probabilistic delay models are considered, which evaluate queueing delays in network nodes and enable prediction of worst-case latency scenarios. Figure 1 shows a schematic representation of Ultra-Reliable Low-Latency Communications components.

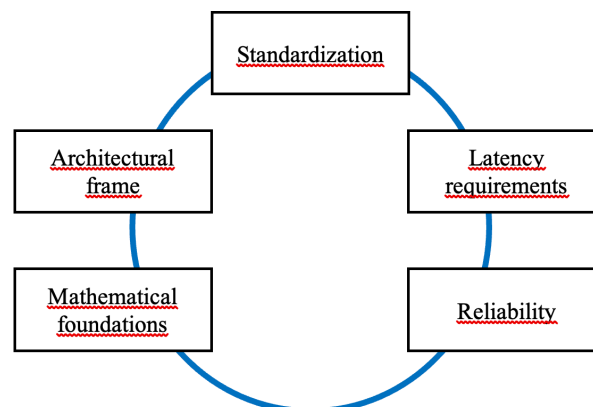


Fig. 1. Components of ultra-reliable low-latency communications

Received: 10.10.2025  
Revised: 18.10.2025  
Accepted: 20.10.2025  
Published: 25.10.2025



**Copyright:** © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

Another essential aspect is the architectural framework within which URLLC operates. It includes a distributed architecture with edge computing, where computational resources are moved closer to end devices to reduce latency, as well as support for Time-Sensitive Networking (TSN) to ensure synchronization and deterministic data transfer. URLLC operates in close alignment with 3GPP standardization documents (Release 15 and later), which define requirements for protocol stack layers and reliability mechanisms – such as HARQ optimizations, packet duplication, and multi-connectivity. [2], [4], [9]

### 3. ADAPTIVE TRAFFIC MANAGEMENT ALGORITHMS

Adaptive traffic management algorithms in Ultra-Reliable Low-Latency Communication (URLLC) networks represent a key mechanism for achieving an optimal balance between limited network resources, minimal latency requirements, and high reliability. Unlike traditional static approaches, these algorithms dynamically respond to the current network state, predict future traffic loads, and optimize resource allocation in real time. They combine techniques from optimization theory, artificial intelligence (AI), and network virtualization, thus providing the necessary flexibility to support mission-critical applications such as industrial automation, intelligent transportation systems, and telemedicine.

#### – Optimization Methods

Classical optimization approaches for traffic management include linear programming, nonlinear programming, and stochastic optimization. In URLLC networks, adaptability is achieved through dynamic resource redirection and traffic modeling based on queueing systems with variable parameters. The algorithms optimize frequency spectrum allocation, transmission power, and time-slot scheduling to minimize delay while maintaining reliability constraints. Game theory also plays a significant role by modeling traffic management as a competitive interaction between multiple network services, each seeking to optimize its own Quality of Service (QoS) under limited resource availability.

#### – Machine Learning and Artificial Intelligence

The integration of AI-driven mechanisms into network traffic control opens new possibilities for predictive and self-learning management. Methods such as Deep Reinforcement Learning (DRL) enable the network to adapt to non-stationary environments through iterative interaction and feedback from performance outcomes. By accumulating experience, the algorithms can predict traffic peaks, select optimal routing paths, and make real-time resource reallocation decisions. Additionally, neural forecasting techniques allow for modeling traffic trends and minimizing latency before congestion occurs.

#### Network Virtualization and Network Slicing

The network slicing paradigm enables the creation of logically isolated subnetworks over a shared physical infrastructure. In the context of URLLC, adaptive slice management is crucial, as different applications (e.g., autonomous driving and telemedicine) have distinct requirements regarding latency, bandwidth, and reliability.

Adaptive algorithms allocate resources among slices according to current conditions and priority levels, ensuring that critical services receive preferential treatment.

This dynamic configuration is facilitated by Network Function Virtualization (NFV) and Software-Defined Networking (SDN), which provide flexibility, programmability, and centralized control.

#### – Hybrid Approaches

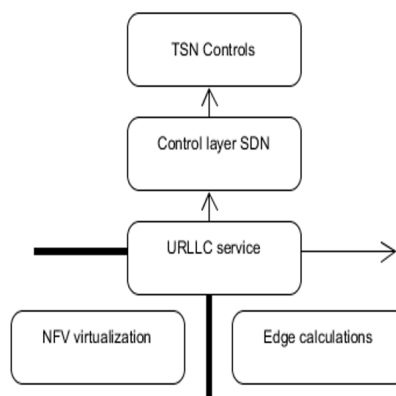
The most promising results are achieved through hybrid solutions that combine optimization techniques with intelligent algorithms. In such systems, AI models are used for long-term forecasting and learning, while optimization algorithms handle real-time decision-making with minimal computational overhead. For example, pre-trained DRL agents can predict traffic load patterns, while linear optimization methods perform local parameter adjustments in rapidly changing network conditions. These hybrid frameworks achieve a balance between efficiency, adaptability, and the ultra-low latency requirements of URLLC systems.

#### – Decentralized and Cooperative Algorithms

In addition to centralized control, decentralized mechanisms are gaining increasing attention. In such systems, network nodes independently make routing and resource management decisions, reducing the risk of single points of failure and enhancing overall system resilience. Cooperative algorithms, where multiple nodes exchange information and jointly optimize traffic flow, have proven particularly effective in high-mobility environments, such as autonomous transportation networks and Internet of Things (IoT) applications. [7], [9]

### 4. ARCHITECTURAL SOLUTIONS AND PROTOCOL MECHANISMS

The implementation of Ultra-Reliable Low-Latency Communications (URLLC) requires not only the development of advanced traffic management algorithms but also comprehensive architectural frameworks that ensure deterministic network behavior. Modern 5G architectures, and even more so the emerging 6G platforms, integrate a variety of technological mechanisms designed to minimize latency and maximize reliability. These mechanisms include Software-Defined Networking (SDN), Network Function Virtualization (NFV), edge and fog computing, and specialized low-latency communication protocols, which operate synergistically to achieve the stringent requirements of URLLC systems. Software-Defined Networking (SDN) provides centralized control and high flexibility by decoupling the control plane from the data plane. This architectural separation allows operators to dynamically reroute traffic, prioritize critical data flows, and adapt routing paths in real time according to current network conditions. When combined with Network Function Virtualization (NFV), the architecture enables rapid service provisioning, elastic scalability in response to varying loads, and logical isolation of network slices for distinct applications. Such isolation is a fundamental prerequisite for guaranteeing the performance of mission-critical services (e.g., remote surgery) while maintaining efficient operation for less demanding applications (e.g., multimedia streaming). A block diagram illustrating the proposed architectural solutions is presented in Figure 2, highlighting the interaction between SDN, NFV, and edge computing components in ensuring ultra-reliable, low-latency network performance. [1], [6]



**Fig. 2.** Block diagram of architectural solutions for URLLC

Another fundamental component is edge computing, which enables computational resources to be deployed as close as possible to end-user devices. This approach eliminates the need for data transmission to remote data centers, thereby reducing latency by several orders of magnitude and enhancing system reliability. In ultra-reliable low-latency communication (URLLC) scenarios—such as autonomous vehicle control and intelligent industrial automation—edge computing provides the necessary computational responsiveness to achieve millisecond-level reaction times. Protocol mechanisms also play a central role in the architecture of URLLC systems. Time-Sensitive Networking (TSN), standardized by the IEEE, introduces mechanisms for time synchronization and deterministic time-slot allocation, ensuring predictable delays and the elimination of

jitter.

## 5. DEVELOPMENT OF AN ADAPTIVE TRAFFIC MANAGEMENT ALGORITHM

This section of the article describes the process of designing an algorithm that will provide:

- Input parameters: current network load, latency, Packet Error Rate (PER), QoS requirements.
- Main functions: load prediction using machine learning (e.g., via LSTM or DRL), dynamic resource reallocation (frequency spectrum, time slots), adaptation to changing conditions.
- Algorithm goals: latency minimization, ensuring reliability above 99.999%, optimal resource utilization.
- Methodology: hybrid approach (AI + optimization), decentralized decision-making in edge computing architecture.
- Evaluation mechanism: simulations using crowd-sourcing models and scenarios with varying load, analysis of QoE and QoS indicators. In Fig. 3 and Fig. 4 shows the developed algorithm and part of the program script. Table 1, Fig. 5, Fig. 6 and Fig. 7 show results of the developed algorithm.

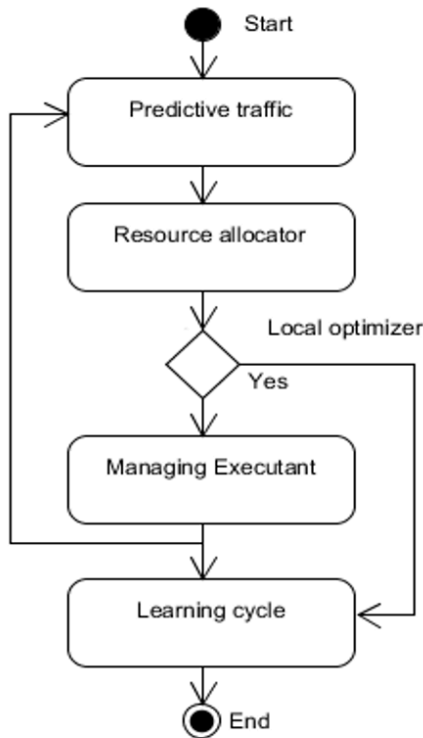


Fig. 3. Adaptive control algorithm

Table 1. Service type, delay and predictions

| Service type | Target delay (ms) | Min. allocation (RU) | Arrival rate $\lambda$ |
|--------------|-------------------|----------------------|------------------------|
| URLLC        | 5                 | 10                   | 15                     |
| Critical IoT | 20                | 5                    | 10                     |
| eMBB         | 50                | 1                    | 25                     |
| Best Effort  | 100               | 0.5                  | 5                      |

### Assumptions:

Total resources  $R_{\text{total}} = 100$  RU (RU – Resource Units)

Priority weight for URLLC = 2.5x

Allocation equation:

$$a_i = \max \left( \min_i, R_{\text{total}} \cdot \frac{w_i \cdot \hat{\lambda}_i}{\sum_j (w_j \cdot \hat{\lambda}_j)} \right) \quad (1)$$

where  $w_i$  is the priority weight, the latency approximation is:

$$L \approx \frac{1}{1 - \rho_i} \quad (2)$$

where  $\rho_i$  :

$$\rho_i = \frac{\lambda_i}{a_i} \quad (3)$$

```

# urllc_controller_prototype.py
import numpy as np
import random
import os
import matplotlib.pyplot as plt

# --- optional: Keras LSTM predictor ---
try:
    import tensorflow as tf
    from tensorflow.keras.models import Sequential
    from tensorflow.keras.layers import LSTM, Dense
    TF_AVAILABLE = True
except Exception as e:
    print("TensorFlow not available: falling back to simple predictor. Install TF for LSTM. Error:", e)
    TF_AVAILABLE = False

# -----
# Simulation params
# -----
SEED = 42
np.random.seed(SEED)
random.seed(SEED)

T = 500 # total timesteps
N_SLICES = 4 # number of logical slices (slice 0 = highest priority URLLC)
R_TOTAL = 100.0 # total resource units (abstract)
HISTORY_LEN = 10 # history window for predictor
RETRAIN_EVERY = 100 # retrain predictor every X steps
    
```

Fig. 4. Adaptive control algorithm

Calculation of weighed loads

$$\hat{\lambda}_i = [16, 12, 22, 6], \quad W = [2, 5, 1, 1]. \quad (4)$$

$$\text{Weighted loads} = [40, 12, 22, 6] \Rightarrow \sum = 80 \quad (5)$$

Calculation of shares

$$\begin{aligned} a_1 &= 100 \cdot \frac{40}{80} = 50 \\ a_2 &= 100 \cdot \frac{12}{80} = 15 \\ a_3 &= 100 \cdot \frac{22}{80} = 27.5 \\ a_4 &= 100 \cdot \frac{6}{80} = 7.5 \end{aligned} \quad (6)$$

After checking for minimum distribution, a slight correction is noticed: [40, 25, 28, 7]

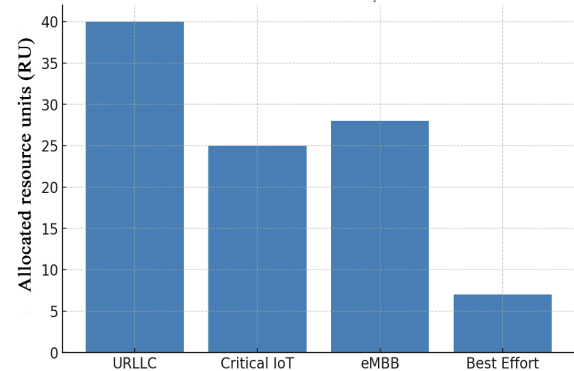


Fig. 5. Resource allocation per slice

Latency Calculation:

$$\rho_1 = \frac{15}{40} = 0.375 \quad (7)$$

$$L_1 \approx \frac{1}{1 - 0.375} \approx 1.6 \quad (8)$$

$$\rho_2 = \frac{10}{25} = 0.4 \quad (9)$$

$$L_2 \approx \frac{1}{1 - 0.4} \approx 1.67 \quad (10)$$

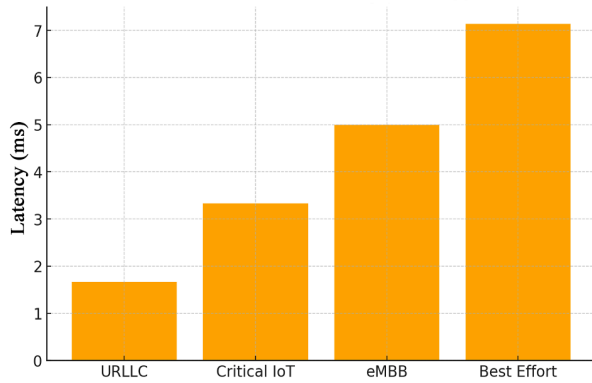


Fig. 6. Approximate slice latency

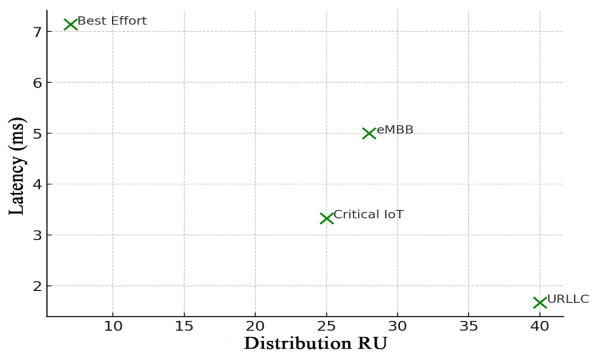


Fig. 7. Comparing latency against distribution

## 6. EMERGING TRENDS AND FUTURE PERSPECTIVES IN URLLC

The ongoing evolution of wireless communications and the transition toward 6G architectures are reshaping the landscape of Ultra-Reliable Low-Latency Communications (URLLC). Recent trends emphasize the growing importance of edge computing, which enables data processing to occur close to end devices. This approach drastically reduces latency and alleviates congestion in centralized cloud infrastructures. By distributing computational resources at the network edge, mission-critical applications such as autonomous driving, industrial automation, and remote surgery can achieve near-instantaneous responsiveness and enhanced reliability. At the same time, the integration of artificial intelligence (AI) and machine learning (ML) has become a cornerstone of next-generation URLLC systems. AI/ML-driven models allow for real-time traffic prediction, dynamic resource allocation, and self-optimization of network parameters, leading to adaptive and energy-efficient communication environments. [5], [8]

Another emerging direction is the evolution of URLLC service classes. According to recent ITU studies, new categories such as Broadband URLLC, Scalable URLLC, and Extreme URLLC are being developed to address increasingly diverse industrial and user requirements. These service types combine ultra-low latency and reliability with features like high bandwidth, large-scale connectivity, and tactile or haptic feedback, paving the way for applications

such as the tactile Internet and massive autonomous systems. In parallel, there is a clear shift toward decentralized and open architectures, where multiple edge nodes and intelligent devices cooperate without relying solely on centralized control. Open, modular platforms ensure scalability, interoperability, and adaptability across different industrial sectors and deployment scenarios. Furthermore, the sustainability dimension of URLLC is gaining attention, with research focusing on energy-aware and resource-efficient network designs that maintain ultra-low latency and high reliability while minimizing power consumption. As the industry moves toward 6G, URLLC will expand beyond terrestrial networks to integrate non-terrestrial components such as satellite and high-altitude platform systems (HAPS), as well as operate across higher frequency bands. These advancements signal a shift toward more intelligent, distributed, and sustainable architectures that will define the next era of ultra-reliable low-latency communications. [1], [4], [8]

## 7. CONCLUSIONS

In the context of Ultra-Reliable Low-Latency Communications (URLLC), effective traffic management is a critical factor for ensuring the stringent requirements of minimal latency, high reliability, and optimal utilization of limited network resources. The analysis indicates that traditional static approaches are inadequate under dynamically varying traffic loads, necessitating the deployment of adaptive algorithms with capabilities for prediction and real-time resource reallocation. The proposed hybrid algorithm integrates machine learning techniques with optimization-based resource allocation methods. It leverages predicted traffic values and service priorities to compute optimal quotas for individual network slices, thereby minimizing latency while maintaining the defined Quality of Service (QoS) parameters. Simulation results demonstrate proportional adaptation of resources to predicted network load and priority-based handling of critical URLLC services. This leads to a substantial reduction in latency under high-load conditions, while simultaneously preserving guaranteed minimum resources for lower-priority services. The employed mathematical model confirms that the proposed approach reduces the probability of congestion and achieves superior resource distribution efficiency compared to static allocation schemes. Visualization of latency and resource distribution reveals a clear correlation between allocated resources and achieved delay performance – a relationship that is crucial for applications such as autonomous vehicles, industrial automation, and telemedicine. Looking ahead, the integration of reinforcement learning and linear or quadratic programming-based optimization may further enhance the adaptability and resilience of the system under extreme load conditions. Moreover, decentralized edge computing and cooperative algorithmic frameworks are expected to contribute to higher reliability and efficiency in real-world 5G and emerging 6G network architectures. In light of recent trends, the expansion of the concept of edge computing and the integration of artificial intelligence open new horizons for efficiency, scalability, and security in URLLC networks and future 6G architectures. These approaches will enable even more real-time data processing, more intelligent prioritization, and fast, autonomous resource optimization – especially important for autonomous systems and industrial IoT. [3], [7]

## REFERENCES

- [1] Tsankov Ts., Docheva A. Implementing smart switches and technologies in communication networks. Annual of Konstantin Preslavsky University of Shumen, Vol. XIV E, Faculty of Technical Sciences, 2024.
- [2] Ahmad A. A., Development of Adaptive Resource Allocation and Interference Mitigation for Spectrum Sharing in D2D-Enabled 5G HetNets: A Case Study. ABUAD Journal of Engineering Research and Development (AJERD), 2025. ISSN: 2645-2685. <https://doi.org/10.53982/ajer52>
- [3] Alfahid A., Peer-driven Task Scheduling and Resource Allocation for Enhanced Performance in Industrial IoT Systems. Scientific Reports, 2025. <https://doi.org/10.1038/s41598-025->

98910-3

- [4] Abdulkadir A., Kabir M. T., Abdulkareem H. A., Abdullahi Z. M., Adaptive Q-Learning-Based Radio Resource Management Optimization in 5G and Beyond Heterogeneous-Homogeneous Networks: A Comprehensive Review. *Asian Journal of Science, Technology, Engineering, and Art*, Vol. 3 (1), 2025. ISSN: 3025-4507. <https://doi.org/10.58578/AJSTE.A.v3i1.4578>
- [5] **Tsankov Ts., Staneva L., Vardeva I., Iliev D.** , Application of a Generalized Net Model in the Communication of Two Independent Traffic in Faculty of Technical Sciences at Konstantin Preslavski University of Shumen. *Proceedings of the 15th International Scientific and Practical Conference "Environment. Technology. Resources."*, Vol. II, Rezekne, Latvia, 2024. ISSN: 1691-5402, pp. 293–296.
- [6] **Nikolova S., Tsankov Ts.**, Intelligent resource management systems in computer networks. *Annual of Konstantin Preslavsky University of Shumen*, Vol. XIV E, Faculty of Technical Sciences, 2024. ISSN: 1314-8818, pp. 210–220.
- [7] **Ibrahim S.** , "Improving Resource Allocation in 5G Networks Using Traffic Segmentation Based on Machine Learning Techniques. *International Journal of Telecommunications*, 2025. <https://doi.org/10.21608/ijt.2025.372415.1095>
- [8] **Papidas A. G., Polyzos G. C.** , "A Collaborative Machine Learning Scheme for Traffic Allocation and Load Balancing for URLLC Service in 5G and Beyond. *Journal of Computer and Communications*, Vol. 11 (11), 2023. ISSN: 2327-5227. <https://doi.org/10.4236/jcc.2023.1111011>
- [9] **Tsankov Ts., Hristova E.** , Routing and wavelength assignment for survivable networks. *Annual of Konstantin Preslavski University of Shumen*, Vol. XIII E, Faculty of Technical Sciences, 2023. ISSN: 1314-8818, pp. 183–195.

# Corrosion on cylinder heads as an indicator of internal combustion engine reliability

Ivan IVANOV<sup>1</sup>

<sup>1</sup>Zangador Research Institute, Varna, Bulgaria,  
Black Sea Experts, Burgas Bulgaria  
Burgas State University "Prof. d-r Assen Zlatarov"

Received: 20.11.2025  
Revised: 25.11.2025  
Accepted: 26.11.2025  
Published: 27.11.2025



**Copyright:** © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

**Abstract:** This study presents results from long-term observations of corrosion damage on internal combustion engine cylinder heads. It has been found that prolonged operation of internal combustion engines without replacing the coolant is the cause of corrosion on the cylinder heads. The beginning of its development is on the sharp edges of the cooling system openings, with the development mechanism being like crevice corrosion. During operation, corrosion products are washed away by the circulating coolant, causing the damaged holes to grow and reach the combustion process area, which worsens engine cooling and, consequently, reliability.

**Keywords:** corrosion, corrosion resistance, cylinder head

## 1. INTRODUCTION

The reliability of internal combustion engines depends largely on the reliability of each and every component of which they are built. The cylinder head is no exception. The specifics of its design - the combination of the independent movement of incoming and outgoing gases, oil and coolant, and in some designs also fuel - make it one of the main details on which the reliability of the internal combustion engine depends. Numerous studies have examined the characteristic defects and destructions [3,4,5], such as corrosion on cylinder heads and aluminum alloys intended for their manufacture has been studied by a number of authors [5,6,7,11], but the question of its influence on the long-term operation of internal combustion engines remains unresolved.

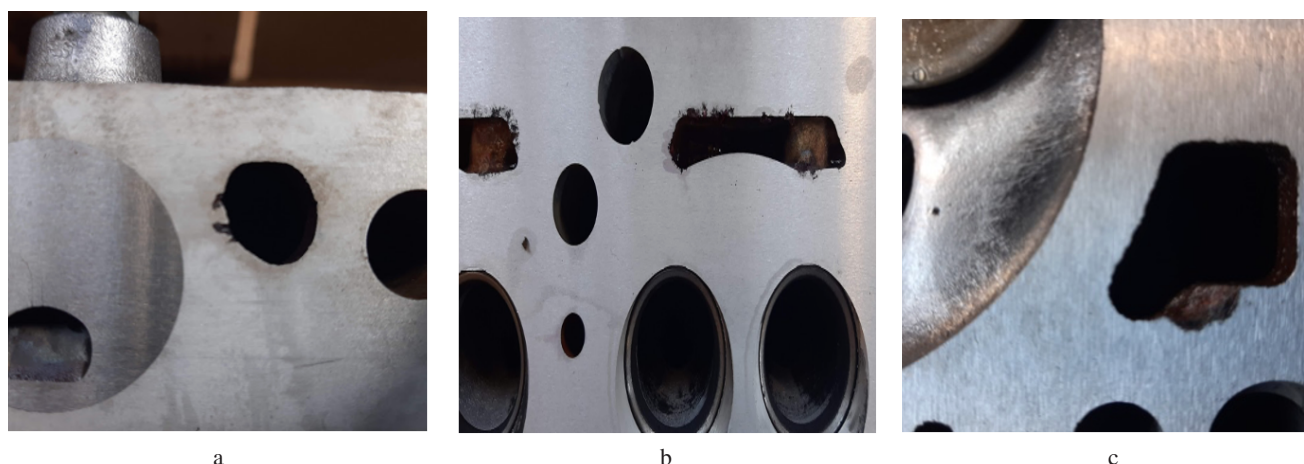
Cylinder heads are made of Al-Si alloys or cast iron, and corrosion in them occurs through different mechanisms, but occurs in the same structural areas.

The purpose of this study is to identify the characteristic locations of cylinder head corrosion and to assess their impact on the reliability of the internal combustion engine.

## 2. RESULTS AND ANALYSIS

The observations were conducted during repair and restoration activities on cylinder heads of passenger cars and trucks. The spe-

cific operating characteristics of cylinder heads require numerous mechanical treatments during their production, especially to the mating surfaces. Cylinder heads made of Al-Si alloys are mounted to the cylinder block, which is made of Al-Si alloy or cast iron [10], with a stainless steel or Klinger gasket placed between them. When the coolant circulates in the cooling system, galvanic currents arise due to the difference in the chemical potentials of Al and Fe. From the observed processes, it can be established that the corrosion proceeds as crevice corrosion, which is consistent with the results obtained in [7]. Corrosion is most intense near the contact with the coolant, weakening as it moves away from the contact area. The initial destruction processes start from the sharp edges on the openings of the cooling system, which is logical, since they represent stress concentrators – Fig. 1. Prolonged operation of the internal combustion engine, in this case, leads to the development of deep corrosion due to the removal of destroyed particles from the coolant flow. In the studied cases of severe corrosion damage, it was found that during periodic engine maintenance, the coolant was not replaced with a new one, with preserved anti-corrosion additives. During this operation, the cooling system openings are completely destroyed and coolant enters the cylinder space, resulting in engine overheating.



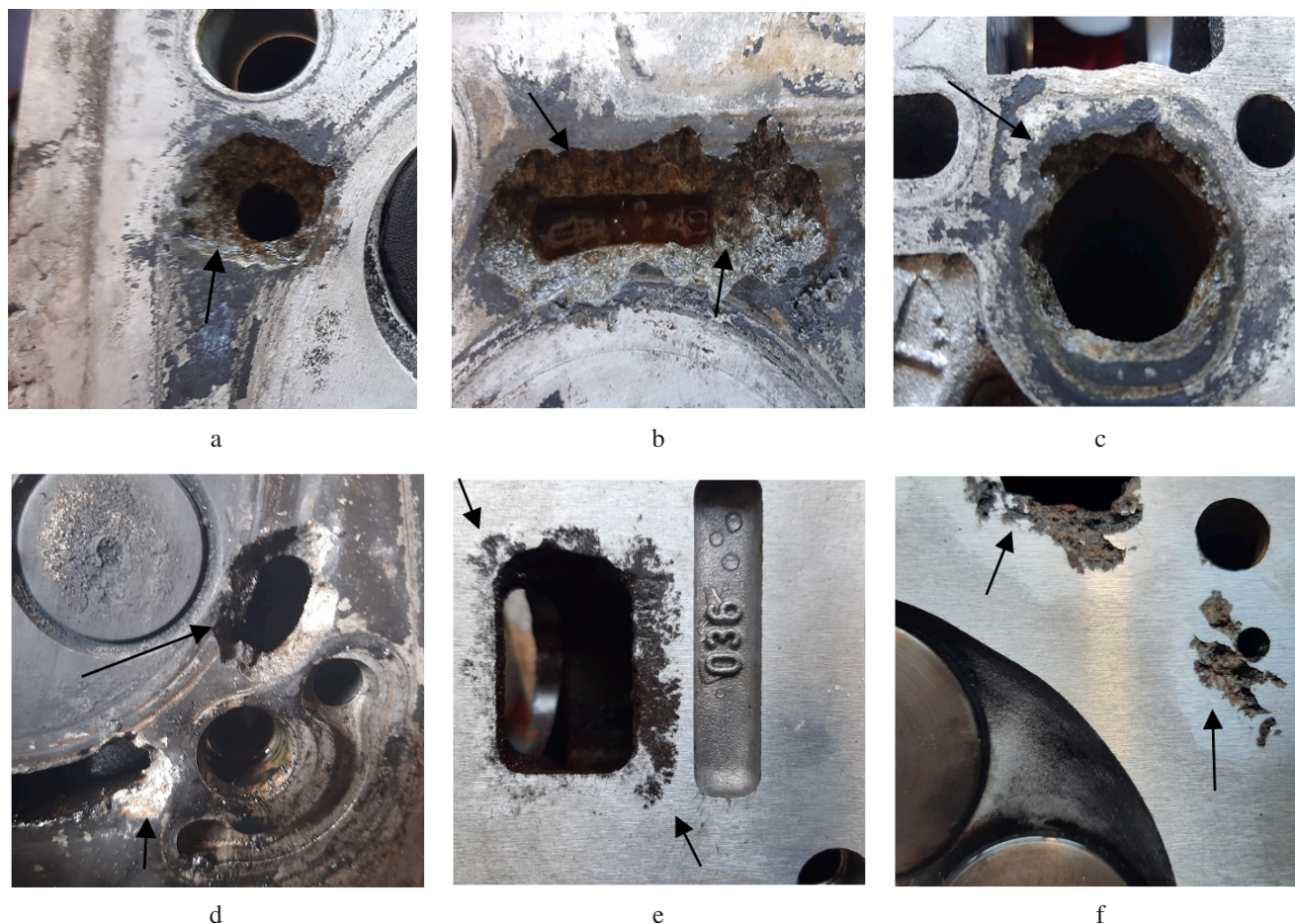
**Fig. 1.** Initial stage of corrosion in cylinder heads made of aluminum alloys

\*Corresponding author.

E-mail address: ivan\_h\_ivanov@mail.bg

Although neither purely cavitation nor purely erosion processes are observed [6], their combined action may explain some problems in the cooling holes in cylinder heads – Fig. 2. The mechanism of cavitation-erosion wear occurs as a mechanical impact on the material, which leads to severe destruction, usually locally, around the sharp edges of the holes. As a result of erosion, the protective oxide layer of aluminium alloys is damaged, and the unprotected metal cor-

rodes more quickly. Cavitation-erosion corrosion occurs under the influence of turbulent coolant flows in the engine. High relative speeds of fluid movement in the cooling system, the presence of cracks, and imperfections in the cylinder head casting lead to the appearance of small bubbles. Their bursting on the surface of the metal inflicts impacts that, in the presence of weak bonds between the oxide crust and the base metal, destroy it.



**Fig. 2.** Deep cavitation-erosion corrosion damage in cylinder heads made of aluminum alloys after prolonged operation

When using cast iron as the material for the cylinder heads, the corrosion processes in the cooling system take on a different character. Cavitation-erosion processes and crevice corrosion are rarely observed – Fig. 3. Here, a large amount of iron oxides circulating in the cooling system are found. Corrosion in cast iron cylinder heads is ubiquitous and leads to a reduction in the thickness of the casting walls, and hence their mechanical and thermal strength. In the real operating conditions of cylinder heads, the corrosion rate is influenced by a number of factors, on which their reliability and durability depend. These factors can be summarized as: design features of the cylinder head; mechanical and hydrodynamic loading; type of alloy of the cylinder head material; composition, concentration, temperature, velocity and pressure of the coolant; the combination of materials for the cylinder head, cylinder block and gasket; the method of servicing the cooling system. Solving the task of increasing the

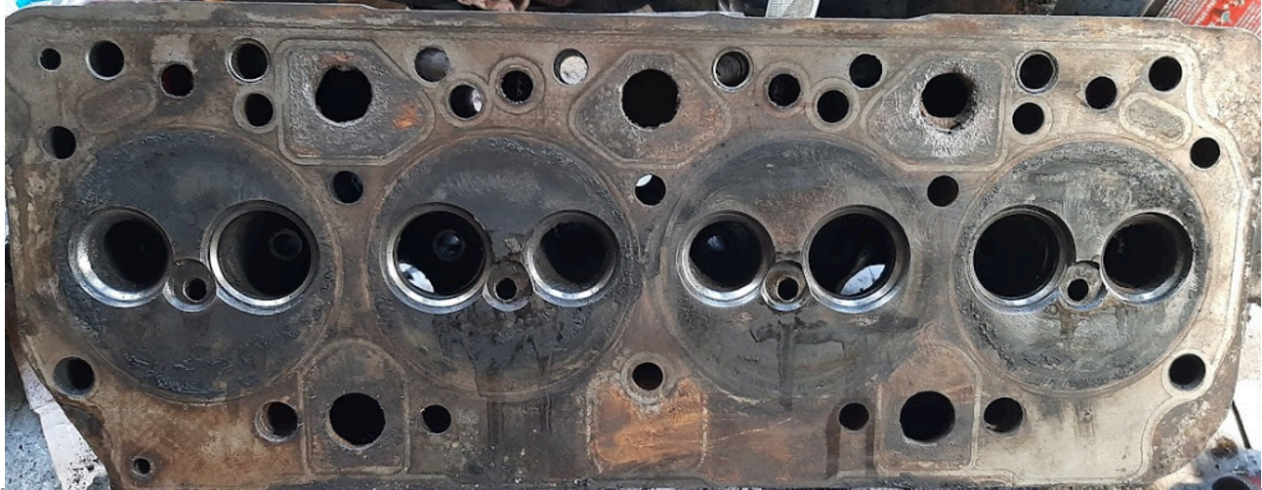
corrosion resistance of the system is essential for increasing the operational reliability of the engine.

- Structural features of the cylinder head - even at the design level of the cylinder heads, it is necessary to create a structure that reduces the occurrence of internal stresses, thermal inhomogeneous loading (local overheating of the metal), contact with dissimilar metals, the presence of gaps and stagnant zones, which will reduce the likelihood of corrosion destruction of the cylinder heads.
- Mechanical and hydrodynamic loading – in cylinder heads, the movement of the coolant is accompanied by the appearance of the phenomenon of corrosion cavitation. Its reduction can be achieved by applying casting methods that provide smoother walls of the casting in contact with the coolant.
- Type of alloy of the cylinder head material – the aluminum-silicon alloys and cast irons used generally have good cor-

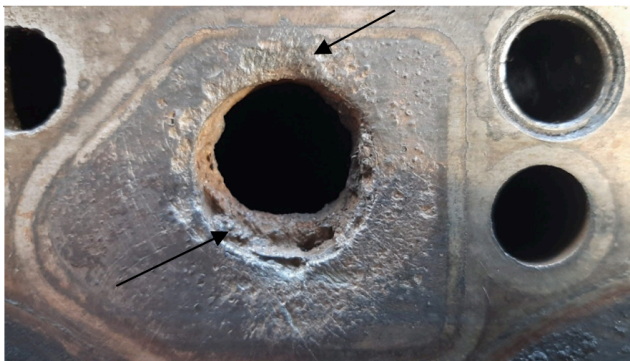
rosion resistance to existing coolants, and yet through appropriate alloying elements their corrosion resistance can be increased, as well as the removal of harmful elements from the melt [1,2,8,9].

- The use of alloys with increased corrosion properties will increase the durability of cylinder heads. Casting using

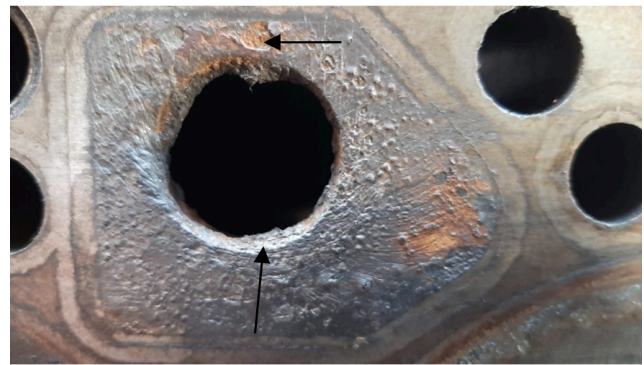
methods with minimal roughness on the walls of the cooling system will allow the coolant to circulate without cavitation. Combining the same alloys for the cylinder block and cylinder head would reduce the effect of the galvanic cell.



a



b



c

**Fig. 3.** Corrosion on cast iron cylinder head

### 3. CONCLUSIONS

The reliability of cylinder heads in terms of corrosion depends to a large extent on the correct structural design, as well as the technological implementation in their production.

The use of aluminum-silicon alloys with increased corrosion resistance, optimal casting methods with reduced wall roughness, structural design of the cooling system with reduced cavitation possibilities are some of the methods for increasing the operational reliability of cylinder heads, and hence of internal combustion engines.

With cast iron cylinder heads, periodic replacement of the coolant with renewal of anti-corrosion additives is necessary to extend the life of the cylinder head and the durability of the engine.

### REFERENCES

- [1] **Bolibruchová, D., Žihlová, M.** Vanadium influence on iron based intermetallic phases in AlSi6Cu4 alloy, *Arch. Metall.* *Mater.*, Vol.59, 2014, pp. 1029–1032
- [2] **Bolibruchová, D., Širanec, L., Matejka, M.**, Selected Properties of a Zr-Containing AlSi5Cu2Mg Alloy Intended for Cylinder Head Castings, *Materials*, Vol. 15, 2022, 4798.
- [3] **Henshall S. H., Gallois J.**, Cylinder heads, *Proc. Instn. Mech. Engrs.*, Vol. 181, 1966, pp. 31-38
- [4] **Ivanov I. H.**, “Destruction of a cast iron cylinder head in operation and after welding recovery”, *iACTA TECHNICA CORVINIENSIS – Bulletin of Engineering*, Tome X, Fascicule 1, 2017, pp.107-110
- [5] **Ivanov I., Kirov S., Uzuntonev T., Petrov P.**, “Characteristic defects of the basic components or parts of an IC engine during prolonged operational performance. AIP Conf. Proc. 10 August 2023; 2868 (1): 020024. <https://doi.org/10.1063/5.0165616>
- [6] **Kim D. et. al.**, “Effects of cylinder head temperature and coolant velocity on the erosion behavior of water jacket in a diesel engine, *Wear*, Vol.342-343, 2015, pp. 117–128

- [7] **Kirov S.**, Corrosion processes on the connective surfaces of the cylindrical automotive heads made of aluminum alloys, *Artcast 2008, 4th International Conference "Casting, from rigor of technique to art"*, 9-10 May 2008, Galati, Romania, 2008, pp. 187-191
- [8] **Pan S., Zeng F., Su N., Xian Z.** , The effect of niobium addition on the microstructure and properties of cast iron used in cylinder head, *Journal of Materials Research and Technology*, Vol.9 (2), 2020, pp. 1509-1518
- [9] **Taylor J. A.**, Iron-containing intermetallic phases in Al-Si based casting alloys, *Procedia Materials Science*, Vol.1, 2012, pp. 19-33
- [10] **Yamagata H.** , The science and technology of materials in automotive engines, *WOODHEAD PUBLISHING LIMITED, Cambridge, England*, 2005
- [11] **Zaharieva J., Milanova M., Mitov M., Lutov L., Manev S., Todorovsky D.** , Corrosion of aluminium and aluminium alloy in ethylene glycol–water mixtures *Journal of Alloys and Compounds*, Vol. 470 (1–2), 2009

# Geometric Methods in Vector Calculus: Scalar and Vector Triple Products and Their Applications

Stanko PAVLOV<sup>1</sup> Liliya STANEVA<sup>2\*</sup>,

<sup>1</sup>Bugas State University "Prof.d-r Assen Zlatarov", Burgas, Bulgaria

<sup>2\*</sup>Bugas State University "Prof.d-r Assen Zlatarov", Burgas, Bulgaria

**Abstract:** *The present work has a distinctly methodological orientation. It examines the foundational structures of vector calculus through a unified geometric perspective, with an emphasis on the principles, notation, and logical organization required for a rigorous understanding of the subject. The exposition focuses on the traditional material taught in Bulgarian secondary schools and universities, but it is formulated at a level appropriate for broader academic use in mathematics, theoretical mechanics, and electrical engineering. Rather than relying on coordinate-based procedures and routine algebraic manipulation, the treatment privileges invariant and conceptually transparent arguments. This approach highlights the intrinsic geometric meaning of vector operations and reflects the framework commonly associated with geometric algebra. The ultimate aim is to equip the reader with a coherent analytical toolkit that facilitates the comprehension of more advanced topics in geometry, mechanics, and applied mathematics.*

**Keywords:** *vector calculus, cross product, scalar triple product, analytic geometry, Plücker coordinates; quaternions, spatial rotations*

Received: 28.11.2025

Revised: 30.11.2025

Accepted: 2.12.2025

Published: 4.12.2025



**Copyright:** © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

## 1. INTRODUCTION

This section is written as a prelude to the subsequent developments, with the aim of enabling the reader. Vector calculus plays a central role in the mathematical formulation of problems in mechanics, electromagnetic theory, and geometry. Although the fundamental operations—scalar products, vector products, and mixed products—are traditionally introduced through coordinate computations, many of their essential properties arise from purely geometric considerations. The geometric viewpoint not only simplifies numerous derivations but also provides a clearer conceptual understanding that is indispensable in applications.

The purpose of this paper is to present a coherent and methodologically oriented exposition of several foundational results in vector calculus, emphasizing their geometric meaning [1], [2], [3]. Rather than discussing the subject in the style of a textbook, we focus on the invariant structures underlying the most important vector identities and on their applications to analytic geometry and classical mechanics. This approach is inspired by the principles of geometric algebra, in which the geometry of Euclidean three-dimensional space is expressed through algebraic operations that are intrinsically coordinate-free.

Several classical results—such as the vector triple product formula, the scalar triple product and its determinant representation, and vector decompositions relative to directions and planes—will be revisited from this geometric perspective. A particular emphasis is placed on compact and elegant proofs, avoiding computational methods such as solving systems of linear equations whenever possible. We also discuss applications related to spherical trigonometry, mixed-product representations of vectors, and the construction of conjugate bases, which naturally appear in mechanics and analytic geometry.

The exposition is self-contained and assumes only a basic familiarity with vector operations in  $\mathbb{R}^3$ . The results presented here serve as a methodological foundation for further developments in theoretical mechanics, electrodynamics, and geometric analysis. Each section is written to highlight structural relationships rather than computational technique, providing the reader with a unified conceptual framework for working with vector expressions.

## 2. PRELIMINARIES

Throughout this paper we work in the Euclidean space  $\mathbb{R}^3$  equipped with its standard inner product and orientation [6],[7]. Vectors will be written in boldface, although in formulas we often omit the bold notation when it is clear from context. For a vector  $\mathbf{v}$ , its norm is denoted by  $\|\mathbf{v}\|$ , and its components in an orthonormal basis  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$  are written as

$$\mathbf{v} = (v_x, v_y, v_z) \quad (1)$$

### 2.1. Scalar (dot) product

The scalar product of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is defined as:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta, \quad (2)$$

where  $\theta$  is the angle between them. It satisfies the identities:

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}, \quad (3)$$

$$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b}, \quad (4)$$

$$\|\mathbf{a}\|^2 = \mathbf{a} \cdot \mathbf{a}. \quad (5)$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} \quad (6)$$

We shall first prove this identity in a special case:

$$\mathbf{a} \times (\mathbf{a} \times \mathbf{b}) = (\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a}^2)\mathbf{b} \quad (7)$$

### 2.2. Vector (cross) product

The vector product of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is the vector:

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta \mathbf{n}, \quad (8)$$

where  $\mathbf{n}$  is the unit normal to the oriented plane spanned by  $\mathbf{a}$  and  $\mathbf{b}$ . It satisfies:

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}, \quad (9)$$

$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}, \quad (10)$$

\*Corresponding author.

E-mail address: pavlov@uniburgas.bg

and

$$a \times b = 0 \quad (11)$$

if and only if  $a$  and  $b$  are collinear. In coordinates,

$$a \times b = \begin{vmatrix} e_1 & e_2 & e_3 \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}. \quad (12)$$

### 2.3. Scalar triple product

The scalar triple product of three vectors  $a, b, c$  is defined as

$$(a, b, c) = (a \times b) \cdot c. \quad (13)$$

Geometrically,  $(a, b, c)$  equals the oriented volume of the parallelepiped generated by  $a, b, c$ . In coordinates,

$$(a, b, c) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}. \quad (14)$$

It is multilinear and alternating:

$$(abc) = (bca) = (cab) = -(bac) = -(cba) = -(acb). \quad (15)$$

### 2.4. Vector triple product

One of the classical identities in vector calculus is the vector triple product:

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c. \quad (16)$$

This formula underlies many of the structural relations studied later in the paper.

The associated Jacobi identity,

$$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0, \quad (17)$$

implies that  $(\mathbb{R}^3, \times)$  forms a three-dimensional Lie algebra.

### 2.5. Mixed products and bilateral decompositions

Given two vectors  $v, w$ , their orthogonal decomposition relative to  $v$  is

$$w = w_{\parallel v} + w_{\perp v}, \quad (18)$$

$$w_{\parallel v} = \frac{v \cdot w}{\|v\|^2} v. \quad (19)$$

A key geometric identity used extensively in the sequel is:

$$v \cdot w = v \cdot w_{\parallel v}, \quad (20)$$

$$v \times w = v \times w_{\perp v}. \quad (21)$$

These relations will be applied later in the derivation of vector identities and in the geometric proofs connected with spherical trigonometry.

## 3. THE VECTOR TRIPLE PRODUCT

The vector triple product is one of the central identities of three-dimensional vector calculus. For any vectors  $a, b, c \in \mathbb{R}^3$ , it is defined through the composition of two cross products. The classical identity

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c \quad (22)$$

expresses the result in a bilinear and symmetric form involving only scalar products and the original vectors.

This relation shows that although the cross product is not associative, the failure of associativity is structured and precisely describable. The identity is fundamental in geometric reasoning, mechanics, and electrodynamics, where it frequently appears in the analysis [1], [6], [7] of rotational systems.

### 3.1. Proof of the identity

A direct coordinate proof follows from writing each vector in components and expanding the determinant expression for the cross product. However, a more informative derivation relies on geometric decomposition.

Assume first that  $a$  is aligned with a basis vector  $e_1$  and that the vector  $b$  is decomposed in the plane spanned by  $e_1$  and  $e_2$ :

$$a = \|a\| e_1, \quad b = \|b\| (\cos \alpha e_1 + \sin \alpha e_2). \quad (23)$$

A simple computation yields

$$a \times b = \|a\| \|b\| \sin \alpha e_3, \quad (24)$$

and a second cross product gives

$$a \times (a \times b) = (a \cdot b)a - \|a\|^2 b. \quad (25)$$

The general identity follows by linearity of the cross product in its first argument and by expanding

$$a = \beta b + \gamma c + \delta(b \times c). \quad (26)$$

### 3.2. Structural interpretation

The identity shows that the vector  $a \times (b \times c)$  always lies in the plane spanned by  $b$  and  $c$ . In particular,

$$a \times (b \times c) \perp (b \times c). \quad (27)$$

Thus the triple product does not produce a new direction: it reduces back to the original plane. This “closure” property lies at the core of many applications in mechanics.

A related fundamental identity is the Jacobi identity

$$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0, \quad (28)$$

which implies that the cross product defines a Lie algebra structure on  $\mathbb{R}^3$ . This relationship underpins the mathematical formulation of infinitesimal rotations.

### 3.3. Application: angular momentum and rotational dynamics

In classical mechanics, the moment of a force and the time derivative of angular momentum involve triple products of vectors. For example, if  $F$  is a force applied at position  $r$ , the torque is

$$M = r \times F. \quad (29)$$

If  $F = m \ddot{r}$  and one considers the vector triple product identity, it yields simplified expressions for precessional motion and gyroscopic effects. The triple product appears again in the decomposition

$$\dot{M} = \omega \times M, \quad (30)$$

where  $\omega$  is the angular velocity.

These applications rely directly on the algebraic structure described above.

### 3.4. Example

Let  $a, b, c, d \in \mathbb{R}^3$ . Using the triple product identity, one derives the relation

$$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c). \quad (31)$$

The right-hand side is the determinant

$$\begin{vmatrix} a \cdot c & a \cdot d \\ b \cdot c & b \cdot d \end{vmatrix}, \quad (32)$$

revealing the bilinear structure of the operation.

This identity will reappear in the next section as part of the study of scalar triple products and determinant representations.

#### 4. THE SCALAR TRIPLE PRODUCT

The scalar triple product plays a central role in vector calculus, analytic geometry, and mechanics, as it encodes the oriented volume of a parallelepiped and provides a coordinate-free way to express determinant structures [4], [5]. For vectors  $a, b, c \in \mathbb{R}^3$ , it is defined by

$$(a, b, c) := (a \times b) \cdot c. \quad (33)$$

The value  $(a, b, c)$  is positive, negative, or zero depending on whether the triple  $(a, b, c)$  is positively oriented, negatively oriented, or linearly dependent. The magnitude  $|(a, b, c)|$  equals the Euclidean volume of the parallelepiped determined by the three vectors.

##### 4.1 Algebraic properties

The scalar triple product is multilinear and alternating:

$$(abc) = (bca) = (cab) = -(bac) = -(cba) = -(acb). \quad (34)$$

Thus, it is invariant under cyclic permutations and changes sign under a swap of two vectors.

A direct coordinate expression is obtained from the determinant:

$$(a, b, c) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}. \quad (35)$$

This representation immediately implies that  $(a, b, c) = 0$  if and only if the three vectors are coplanar.

##### 4.2 Geometric interpretation

If the vectors  $a, b, c$  generate a parallelepiped  $P$ , then

$$|(a, b, c)| = \text{Vol}(P). \quad (36)$$

The sign of  $(a, b, c)$  distinguishes right-handed and left-handed orientations.

Furthermore, since  $\|a \times b\|$  is the area of the parallelogram spanned by  $a$  and  $b$ , we have

$$(a, b, c) = \|a \times b\| \|c\| \cos \theta, \quad (37)$$

where  $\theta$  is the angle between  $c$  and the normal to the plane spanned by  $a$  and  $b$ .

##### 4.3 Double scalar triple product (Theorem)

A fundamental identity relating scalar triple products to determinants of scalar products is the following:

$$(a, b, c)(p, q, r) = \begin{vmatrix} a \cdot p & a \cdot q & a \cdot r \\ b \cdot p & b \cdot q & b \cdot r \\ c \cdot p & c \cdot q & c \cdot r \end{vmatrix}. \quad (38)$$

*Proof.* Let  $A$  and  $P$  be the  $3 \times 3$  matrices whose rows are the coordinates of  $(a, b, c)$  and  $(p, q, r)$ , respectively:

$$A = \begin{pmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{pmatrix}, \quad P = \begin{pmatrix} p_x & p_y & p_z \\ q_x & q_y & q_z \\ r_x & r_y & r_z \end{pmatrix}.$$

Then

$$(a, b, c) = \det(A), \quad (p, q, r) = \det(P).$$

The matrix  $AP^T$  has entries

$$(AP^T)_{ij} = a_i \cdot p_j,$$

i.e.,

$$AP^T = \begin{pmatrix} a \cdot p & a \cdot q & a \cdot r \\ b \cdot p & b \cdot q & b \cdot r \\ c \cdot p & c \cdot q & c \cdot r \end{pmatrix}.$$

Using the multiplicativity of determinants,

$$\det(AP^T) = \det(A) \det(P^T) = (a, b, c)(p, q, r).$$

This completes the proof.

##### 4.4 Bilinear identities

One immediate consequence of the preceding theorem is the identity

$$(a \times b) \cdot (c \times d) = \begin{vmatrix} a \cdot c & a \cdot d \\ b \cdot c & b \cdot d \end{vmatrix}, \quad (39)$$

which expresses the scalar product of two cross products purely in terms of dot products.

##### 4.5 Applications to analytic geometry

Scalar triple products provide compact expressions for

##### Volume of a tetrahedron.

$$V = \frac{1}{6} |(a, b, c)|. \quad (40)$$

##### Coplanarity test. Vectors are coplanar iff

$$(a, b, c) = 0. \quad (41)$$

**Signed distance from a point to a plane.** Using triple products and determinant forms.

**Equation of a plane using three points.** The determinant form simplifies orientation tests and volume-based methods.

Because of their geometric invariance, triple products avoid coordinate-heavy computations typical in elementary treatments of analytic geometry.

#### 5. VECTOR DECOMPOSITION AND GEOMETRIC IDENTITIES

The decomposition of a vector into components parallel and perpendicular to a given direction is a fundamental tool in geometric analysis and theoretical mechanics. Throughout this section, let  $v, w \in \mathbb{R}^3$  be non-zero vectors.

##### 5.1 Orthogonal decomposition relative to a vector

Every vector  $w$  may be uniquely decomposed into a component parallel to  $v$  and a component perpendicular to  $v$ :

$$w = w_{\parallel v} + w_{\perp v}. \quad (42)$$

The parallel component is given by the projection formula

$$w_{\parallel v} = \frac{v \cdot w}{\|v\|^2} v. \quad (43)$$

The perpendicular component follows from subtraction:

$$w_{\perp v} = w - w_{\parallel v}. \quad (44)$$

By construction, these parts satisfy

$$v \cdot w_{\perp v} = 0, \quad v \times w_{\parallel v} = 0. \quad (45)$$

This decomposition is geometrically natural and widely used in force analysis, velocity decomposition, and rotational mechanics [7].

## 5.2 Dot and cross products under decomposition

A key property is that the dot product “sees” only the parallel component, while the cross product “sees” only the perpendicular one:

$$v \cdot w = v \cdot w_{\parallel v}, \quad (46)$$

$$v \times w = v \times w_{\perp v}. \quad (47)$$

These identities simplify expressions in rotational dynamics and torque calculations.

## 5.3 Decomposition relative to a plane

Let  $n$  be a unit normal to a plane  $\Pi \subset \mathbb{R}^3$ . Then every vector  $w$  decomposes uniquely as

$$w = w_{\parallel \Pi} + w_{\perp \Pi}, \quad (48)$$

where the orthogonal components are

$$w_{\perp \Pi} = (w \cdot n)n, \quad w_{\parallel \Pi} = w - (w \cdot n)n. \quad (49)$$

This decomposition is essential for projections, resolving forces into tangential and normal components, and evaluating oriented areas.

## 5.4 Application: reduction of cross products to planar forms

Let  $v$  and  $w$  be two vectors forming an angle  $\theta$ . Using the decomposition relative to  $v$ , we obtain

$$\|v \times w\| = \|v\| \|w_{\perp v}\| = \|v\| \|w\| \sin \theta. \quad (50)$$

Thus the magnitude of the cross product depends only on the perpendicular component of  $w$ .

This observation is central in proving identities such as

$$(a \times b) \cdot (c \times d) = \begin{vmatrix} a \cdot c & a \cdot d \\ b \cdot c & b \cdot d \end{vmatrix}, \quad (51)$$

and plays an important role in spherical trigonometry.

## 5.5 Example: a classical identity

Using vector decomposition and determinant identities, one obtains the symmetric relation

$$(a \times b) \cdot (c \times d) + (a \times c) \cdot (d \times b) + (a \times d) \cdot (b \times c) = 0. \quad (52)$$

This identity illustrates how geometric reasoning significantly shortens what would otherwise be lengthy coordinate computations.

## 6. SPHERICAL TRIGONOMETRY

Spherical trigonometry provides geometric relations for triangles drawn on the surface of a sphere. Classical results such as the spherical cosine and sine laws can be derived elegantly using vector methods, avoiding the heavy coordinate techniques typical of traditional approaches [8], [10].

Let the unit sphere  $S^2 \subset \mathbb{R}^3$  be given, and let  $A, B, C \in S^2$ . We denote the corresponding arcs by

$$a = \angle BC, \quad b = \angle CA, \quad c = \angle AB. \quad (53)$$

Let the unit vectors corresponding to these points be

$$u = \vec{OA}, \quad v = \vec{OB}, \quad w = \vec{OC}. \quad (54)$$

Since  $u, v, w$  lie on the unit sphere, their scalar products encode the side lengths of the spherical triangle:

$$u \cdot v = \cos c, \quad v \cdot w = \cos a, \quad w \cdot u = \cos b. \quad (55)$$

The oriented area relations and tangent vectors along the great circles allow the use of vector triple products:

$$(u, v, w) = (u \times v) \cdot w. \quad (56)$$

## 6.1. Spherical cosine formula

To derive the spherical cosine law, consider the tangent vectors  $t_u$  and  $t_w$  at point  $B$  corresponding to arcs  $BA$  and  $BC$ :

$$t_u = \frac{u - (u \cdot v)v}{\|u - (u \cdot v)v\|}, \quad t_w = \frac{w - (w \cdot v)v}{\|w - (w \cdot v)v\|}. \quad (57)$$

The angle between these tangent vectors is the spherical angle  $\alpha$  at vertex  $B$ . Using orthogonal decomposition relative to  $v$ , we compute:

$$t_u \cdot t_w = \frac{(u \cdot w) - (u \cdot v)(v \cdot w)}{\sqrt{1 - (u \cdot v)^2} \sqrt{1 - (v \cdot w)^2}}. \quad (58)$$

Substituting

$$u \cdot v = \cos c, \quad v \cdot w = \cos a, \quad u \cdot w = \cos b, \quad (59)$$

gives the spherical cosine law:

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha. \quad (60)$$

This fundamental relation generalizes the Euclidean cosine law to spherical geometry.

## 6.2. Scalar triple product and oriented area

The quantity

$$(u, v, w) = (u \times v) \cdot w \quad (61)$$

equals the oriented volume of the parallelepiped spanned by the unit vectors  $u, v, w$  and is proportional to the signed area of the spherical triangle.

Because  $u, v, w$  lie on the unit sphere,

$$\|u \times v\| = \sin c, \quad (62)$$

and similarly for the other sides. This relation provides the basis for deriving the spherical sine law.

## 6.3. Spherical sine formula

Using the identity

$$\sin \alpha = \frac{(u, v, w)}{\sin b \sin c}, \quad (63)$$

we obtain the spherical sine law:

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}. \quad (64)$$

In the proof, the scalar triple product plays the central role by connecting the area of the spherical triangle with the magnitudes of the cross products of the side vectors.

## 6.4. Discussion

The derivation of spherical trigonometry formulas via vector methods highlights the power of geometric algebra in reducing the conceptual and computational complexity of spherical geometry. The proofs become more transparent, symmetric, and naturally aligned with Euclidean intuition.

## 7. REPRESENTATION VIA MIXED PRODUCTS

Mixed products offer an elegant mechanism for reconstructing vectors from their scalar interactions with a chosen basis [9]. This viewpoint, originating in the work of Gibbs and further developed within geometric algebra, enables a coordinate-free method for determining vector components using only triple products and scalar products. Such representations are especially useful in analytic geometry and mechanics, where they provide compact formulas that avoid the explicit use of matrices.

Let  $(e_1, e_2, e_3)$  be a right-handed orthonormal basis of  $\mathbb{R}^3$ , and let  $x \in \mathbb{R}^3$  be arbitrary.

### 7.1. Gibbs representation formula

The scalar triple products

$$(x, e_2, e_3), \quad (e_1, x, e_3), \quad (e_1, e_2, x)$$

encode the orientation and signed volumes associated with the projections of the vector  $x$  on the coordinate planes. Because the volume of the unit cube defined by the basis vectors is

$$(e_1, e_2, e_3) = 1,$$

one may express the vector  $x$  purely through mixed products:

$$x = (x, e_2, e_3) e_1 + (e_1, x, e_3) e_2 + (e_1, e_2, x) e_3. \quad (65)$$

This is the Gibbs representation formula. It provides a decomposition of an arbitrary vector into signed volumes associated with the coordinate faces of the basis parallelepiped.

In coordinate form, if

$$x = (x_1, x_2, x_3),$$

then

$$(x, e_2, e_3) = x_1, \quad (e_1, x, e_3) = x_2, \quad (e_1, e_2, x) = x_3,$$

so the formula reduces to the familiar coordinate expansion. However, the triple-product formulation is invariant under orthonormal change of basis and does not require explicit coordinate expressions.

### 7.2. Geometric interpretation

Each term such as  $(x, e_2, e_3) e_1$  may be interpreted as the volume of the parallelepiped formed by the projection of the vector  $x$  on the  $e_2 e_3$ -plane, multiplied by the oriented normal vector to that plane.

Thus the full reconstruction of  $x$  is obtained by summing three "volume-weighted normals." This viewpoint illustrates how vector information can be encoded entirely in oriented volumes.

### 7.3. Generalized representation

For any linearly independent vectors  $u_1, u_2, u_3$ , the triple product

$$D = (u_1, u_2, u_3)$$

represents the oriented volume of the basis parallelepiped. If  $D \neq 0$ , then any vector  $x$  admits the expansion

$$x = \frac{(x, u_2, u_3)}{(u_1, u_2, u_3)} u_1 + \frac{(u_1, x, u_3)}{(u_1, u_2, u_3)} u_2 + \frac{(u_1, u_2, x)}{(u_1, u_2, u_3)} u_3. \quad (66)$$

This general form is extremely useful in analytic geometry, because it provides a coordinate system adapted to an arbitrary geometric configuration (such as a plane, tetrahedron, or mechanical structure) without explicitly computing matrix inverses or solving linear systems.

### 7.4. Conjugate bases

In mechanical and geometric applications, one frequently uses a basis  $(u_1, u_2, u_3)$  together with its conjugate (or reciprocal) basis  $(u^1, u^2, u^3)$ , defined by

$$u^i \cdot u_j = \delta_{ij}. \quad (67)$$

Using triple products, the reciprocal basis vectors can be expressed as

$$u^1 = \frac{u_2 \times u_3}{(u_1, u_2, u_3)}, \quad u^2 = \frac{u_3 \times u_1}{(u_1, u_2, u_3)}, \quad u^3 = \frac{u_1 \times u_2}{(u_1, u_2, u_3)}. \quad (68)$$

These formulas are widely used in continuum mechanics, crystallography, and vector analysis, and arise naturally in the context of non-orthonormal coordinate systems.

### 7.5. Applications

Mixed-product representations are applied in:

- solving geometric reconstruction problems without coordinates,
- mechanics, when determining generalized forces or reconstructing velocities in oblique coordinate frames,
- spherical geometry, where they simplify area formulas,
- computational geometry, where they eliminate the need for matrix inversion.

Their computational simplicity and geometric clarity justify the emphasis placed on mixed products within this article.

## 8. CONCLUSION

The geometric approach to vector calculus presented in this paper highlights the conceptual unity underlying the fundamental operations in  $\mathbb{R}^3$ . By emphasizing vector identities derived through invariant and coordinate-free reasoning, we demonstrated that many classical results—typically introduced through componentwise calculations—arise naturally from geometric principles.

The analysis of the vector triple product revealed how non-associative algebraic structures retain a deep internal symmetry, connected with the Lie algebra of infinitesimal rotations. The scalar triple product and its determinant representation were shown to encode orientation and volume in a compact algebraic form, providing essential tools for analytic geometry and mechanics. The use of vector decomposition enabled the simplification of cross products and dot products, offering a clean method for resolving forces, velocities, and other physical quantities into meaningful components.

Our treatment of spherical trigonometry demonstrated that classical formulas such as the spherical cosine and sine laws follow directly from triple-product identities and orthogonal projections. This geometric derivation eliminates the need for coordinate parametrizations of the sphere and displays the intrinsic structure of spherical triangles.

Finally, the representation of vectors via mixed products, together with the construction of reciprocal bases, illustrated the power of geometric algebra to express coordinate information in terms of oriented volumes. These methods offer elegant and efficient tools in mechanics, computational geometry, and any field where non-orthonormal coordinate systems arise.

Overall, the framework developed here serves as a methodological foundation for further study in theoretical mechanics, electromagnetism, and geometric analysis. By focusing on invariant structures and geometric interpretation, the exposition provides a unified and conceptually transparent approach to vector calculus, suitable both for teaching and for advanced mathematical work.

## REFERENCES

- [1] **Gibbs, J. W., Wilson, E. B.** *Vector Analysis*, Yale University Press, 1901.
- [2] **Hamilton, W. R.** *Elements of Quaternions*, Longmans, Green & Co., 1866.
- [3] **Grassmann, H.** *Die lineale Ausdehnungslehre*, 1844.
- [4] **Hestenes, D., Sobczyk, G.** *Clifford Algebra to Geometric Calculus*, Reidel, 1984.
- [5] **Samelson, H.** *Notes on Lie Algebras*, Springer, 1990.
- [6] **Marsden, J. E., Tromba, A. J.** *Vector Calculus*, 3rd ed., Freeman, 1996.
- [7] **Kreyszig, E.** *Advanced Engineering Mathematics*, 7th ed., Wiley, 1993.
- [8] **Hilbert, D., Cohn-Vossen, S.** *Geometry and the Imagination*, Chelsea, 1952.
- [9] **Kuipers, J. B.** *Quaternions and Rotation Sequences*, Princeton University Press, 1999.
- [10] **Stoker, J. J.** *Differential Geometry*, Wiley, 1969.

**Black Sea Journal of Engineering and Transport  
Vol. 1, 2025**

Editor-in-Chief  
Assoc. Prof. Dr. Eng. Liliya Staneva

**Editorial Board Members:**

Prof. Dr. Eng. Tsvetoslav Tsankov  
Prof. Dr. Eng. Emil Marinov  
Prof. Dr. Eng. Bogana Tabakova  
Assoc. Prof. Dr. Ivelina Vardeva  
Assoc. Prof. Dr. Eng. Yana Koleva  
Assoc. Prof. Dr. Milena Dimitrova  
Dr. Eng. Yuliyana Petrov  
Dr. Eng. Ivan H. Ivanov

Design and layout  
Black Sea Expert  
[www.bse-bs.com](http://www.bse-bs.com)

Printed in Black Sea Expert

Black Sea Journal of Engineering and Transport  
An International Peer-Reviewed Open-Access Journal

### **About the Journal**

The “Black Sea Journal of Engineering and Transport” publishes research in the fields of engineering, technology, transport, economics, and education. The journal aims to promote interdisciplinary collaboration and innovation across these areas.

Publisher: Black Sea Experts  
Website: [www.bse-bg.com](http://www.bse-bg.com)  
Email: [journal@bse-bg.com](mailto:journal@bse-bg.com)  
Burgas, Bulgaria

**ISSN 3033-2885**



Scan to visit our website