

Teaching Complex Numbers through Coordinated Representations: an Interactive Lecture Model Integrating Dynamic Visualization

Valentina TODOROVA-LAZAROVA¹

¹Department of Mathematics, Informatics and Natural Sciences,
Technical University of Gabrovo, Gabrovo, Bulgaria

Abstract: Teaching complex numbers in undergraduate mathematics is frequently associated with difficulties in coordinating algebraic, geometric, trigonometric, and exponential representations. Different forms are often treated as unrelated symbolic procedures rather than equivalent descriptions of the same mathematical object, a pattern linked to insufficient coordination between symbolic and geometric reasoning. A structured interactive lecture model for teaching complex numbers using GeoGebra in undergraduate engineering mathematics education is proposed in the present study. The study is grounded in APOS theory and the theory of semiotic representations, and adopts a theoretical-methodological design combining selective literature review with instructional modelling. Learning difficulties associated with complex number instruction were analyzed, the role of dynamic visualization and GeoGebra in supporting conceptual understanding and representational transitions was examined, and directions for future empirical validation were identified. The proposed model organizes instruction as a progressive sequence of transitions between algebraic, geometric, trigonometric, and exponential forms, integrating GeoGebra visualization with guided questioning, prediction tasks, and formative assessment embedded throughout the lecture process. Empirical validation is not included at this stage and represents the primary limitation of the study. Implementation may require adaptation to institutional conditions and heterogeneous student populations. The contribution of the study lies in the systematic integration of dynamic visualization, guided interaction, and representational coordination within a unified lecture design applicable to undergraduate engineering mathematics courses and adaptable for other topics involving multiple representational forms.

Keywords: complex numbers; GeoGebra; representational coordination; dynamic visualization; undergraduate mathematics

1. INTRODUCTION

Complex numbers occupy a central place in undergraduate mathematics and engineering education, given their fundamental role in higher mathematics and a wide range of technical applications. Despite this importance, considerable difficulties are encountered by students in establishing connections between the algebraic, geometric, and trigonometric representations of complex numbers [8, 12, 13, 14]. These difficulties are not merely procedural in nature. Different forms of complex numbers are frequently treated as unrelated symbolic routines, detached from their geometric meaning [12, 13, 14]. Difficulties are reported specifically in the geometric interpretation of modulus and argument, in understanding multiplication as rotation and scaling, and in coordinating symbolic with visual reasoning [8, 13, 14]. As demonstrated by Mutambara and Tsakeni [8], procedural levels of understanding are often maintained without progression toward conceptual interpretation – that is, without complex numbers being constructed as coherent mathematical objects. Dynamic visualization and GeoGebra-supported environments have been shown to facilitate connections between symbolic and geometric forms of complex numbers and to support transitions between representational registers [2, 3, 12, 13, 15]. Positive effects on conceptual understanding and misconception reduction have been reported across multiple studies [3, 15]. However, these effects are not produced automatically by visualization alone. Conceptual understanding depends equally on the organization of instruction, guided classroom interaction, and systematic discussion of representational connections [1, 4, 7, 9, 10]. De-

spite the growing body of research on visualization and technology-supported approaches to complex numbers, a structured interactive lecture model in which algebraic, geometric, trigonometric, and exponential forms are systematically coordinated through guided interaction remains insufficiently developed in the literature. To address this gap, the present study proposes an interactive lecture model grounded in APOS theory and the theory of semiotic representations, integrating dynamic visualization and classroom interaction within a coherent instructional sequence. To achieve this aim, the following objectives are accomplished:

1. To analyze difficulties associated with learning complex numbers in undergraduate mathematics education.
2. To examine the role of dynamic visualization and GeoGebra in supporting conceptual understanding and representational transitions.
3. To develop an interactive lecture structure integrating algebraic, geometric, trigonometric, and exponential representations of complex numbers.
4. To identify instructional elements supporting guided interaction during lecture-based mathematics instruction.
5. To discuss the expected educational implications of the proposed model.
6. To outline directions for future empirical validation.

Received: 11.07.2026

Revised: 02.08.2026

Accepted: 04.08.2026

Published: 11.08.2026



Copyright: © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

*Corresponding author.

E-mail address: v.todorova.lazarova@gmail.com

The paper is organized as follows. First, the theoretical foundations of the model are presented. Next, the proposed interactive lecture structure is described. Finally, the educational implications and potential directions for future research are discussed.

2. THEORETICAL BECKGROUND

The proposed instructional model is grounded in two complementary theoretical perspectives: the theory of semiotic representations [6] and APOS theory [5]. APOS theory is selected because it describes the cognitive progression through which mathematical concepts are constructed from procedural actions toward coherent conceptual structures – directly relevant to the difficulties in representational coordination identified in the preceding section. The theory of semiotic representations is adopted because it provides analytical tools for examining transitions between the multiple representational forms central to the proposed model. Together, these perspectives offer both a cognitive and a representational foundation for the design of the instructional sequence.

Teaching complex numbers through multiple representations

Among the central difficulties in learning complex numbers are transitions between algebraic, geometric, trigonometric, and exponential forms [8, 12, 13, 14]. Rather than being recognized as equivalent descriptions of the same mathematical object, these forms are frequently treated as unrelated symbolic procedures [6, 12, 13]. A particular challenge is posed by the connection between symbolic procedures and geometric interpretation, which often results in fragmented understanding across representational forms. Seloane et al. [12] have demonstrated that coordinated transitions between symbolic and geometric representations contribute to stronger interpretation of mathematical relationships. In a subsequent study [13], GeoGebra-supported modelling activities have been reported to reduce misconceptions by strengthening connections between rectangular, polar, and geometric forms. Working from an APOS-based perspective [5], Mutambara and Tsakeni [7] have shown that procedural levels of understanding are frequently maintained without progression toward object-level construction of complex numbers. Smith et al. [14] have emphasized that successful work with complex algebra depends on the coordination of symbolic and geometric reasoning, with representational fluency closely linked to conceptual understanding. GeoGebra and dynamic visualization Mathematical relationships that are difficult to convey through static representations may be explored and interpreted more effectively within dynamic visualization environments [2, 3]. GeoGebra enables simultaneous display of symbolic and geometric properties, making mathematical relationships directly observable during instruction [3]. Conjecture formulation, property discussion, and interpretation of graphical transformations have been observed among students working with dynamic visualizations during classroom interaction [2]. In the context of complex numbers specifically, connections between symbolic and visual reasoning may be strengthened through real-time visualization of modulus, argument, and algebraic operations [12, 13]. Positive effects on mathematics learning outcomes across multiple instructional contexts have been confirmed by a meta-analysis of GeoGebra-supported instruction [15]. These benefits are not, however, produced automatically. Educational effectiveness depends on instructional organization, classroom interaction, and guided discussion [11]. Superficial interpretation may result when visualization is not accompanied by mathematical explanation. Additional challenges include potential cognitive overload, difficulties in transferring visual observations into formal reasoning, and constraints associated with classroom organization and time management.

Interactive and active learning in undergraduate mathematics

Interaction, active learning, and student engagement constitute distinct but related dimensions of undergraduate mathematics instruction [1, 4, 7, 9, 10]. The organization of classroom communication is the concern of interaction; student participation in cognitive processes is emphasized by active learning; the degree of involvement in learning activities is captured by engagement. Positive

effects on mathematical reasoning and student engagement have been associated with questioning, discussion, conceptual tasks, and technology-supported activities [1, 7]. Responsiveness to student learning needs may be improved through systematic integration of interaction into lecture organization [10], though a balance between participatory approaches and curriculum coverage is required [1]. The stimulating effect of conceptual questioning and classroom voting on mathematical reasoning during undergraduate instruction has been demonstrated empirically [10]. Unresolved challenges are also identified in the literature. Inconsistent conceptualization of student engagement and technology integration across studies has been noted [4], and unified frameworks connecting digital technologies, pedagogy, and mathematical learning processes remain lacking [9].

Theoretical integration of APOS and semiotic perspectives

APOS theory [5] and the theory of semiotic representations [6] are combined within the proposed instructional model. Expected cognitive development during instruction is described through APOS components; the representational shifts students are expected to coordinate are indicated through semiotic transitions. The lecture sequence is organized so that complex numbers are gradually constructed through systematic representational transitions between symbolic, geometric, trigonometric, and exponential forms. The introductory stage is interpreted as a phase of cognitive necessity preceding formal concept construction, in which the need for extension of the number system is motivated. A progressive trajectory is followed in the remaining stages, through which actions on symbolic expressions develop into processes, object-level understanding, and finally a coordinated schema encompassing multiple representational forms. The relationship between lecture stages and theoretical perspectives is presented in Table 1.

Table 1. Relationship between lecture stages and theoretical perspectives

Lecture stage	APOS component	Semiotic transition
Extension of number systems	Pre-action (cognitive necessity)	Symbolic reasoning
Imaginary unit	Action	Symbolic manipulation
Algebraic form	Action → Process	Informal symbolic reasoning → algebraic representation
Complex plane	Process → Object	Algebraic → geometric
Trigonometric form	Object → Schema	Geometric ↔ trigonometric
Operations	Schema	Coordinated algebraic, geometric, and trigonometric representations
Consolidation	Schema	Multiple ↔ coordinated (algebraic, geometric, trigonometric, exponential)
Exponential form	Schema	Trigonometric ↔ exponential

3. METHODOLOGY

A theoretical-methodological research design was adopted in the present study, focused on the development of an interactive lecture model for teaching complex numbers in undergraduate mathematics education. The literature review is analytical and selective rather than fully systematic. Publications were selected according to their relevance to undergraduate mathematics education, complex number instruction, dynamic visualization, GeoGebra-supported learning, and interactive teaching approaches. Priority was given to peer-reviewed studies published predominantly within the last decade and indexed in Scopus or Web of Science, together with foundational theoretical works related to representational coordination and APOS-based perspectives on concept construction. The review served as a theoretical basis for the development of a structured didactic model, not as a systematic meta-analysis. The research design combined analysis of contemporary studies in mathematics education, instructional design modelling of lecture organization, development of instructional sequences supporting coordinated representational transitions, and design of GeoGebra-supported visualization activities. Theoretical synthesis was employed to connect cognitive, representational, and instructional perspectives into a coherent instructional framework. The methodolog-

ical approach proceeded through three interconnected stages. In the first stage, research on conceptual understanding, misconceptions, representational transitions, and visualization in the learning of complex numbers was analyzed [8, 12, 13, 14]. The second stage focused on dynamic visualization, GeoGebra-supported instruction, and interaction between symbolic and geometric representations [2, 3, 11, 15]. In the third stage, an interactive lecture structure was designed, integrating algebraic, geometric, trigonometric, and exponential representations with dynamic interpretation of operations in the complex plane. The lecture model was organized through progressive representational transitions and incorporates guided questioning, prediction tasks, conceptual discussion, and GeoGebra visualization. Empirical validation was not included at this stage of the research. Future studies will involve classroom implementation and pedagogical experimentation investigating conceptual understanding, representational fluency, student engagement, and the effectiveness of the proposed lecture organization in undergraduate mathematics education.

4. INTERACTIVE LECTURE MODEL FOR TEACHING COMPLEX NUMBERS

The structure of the proposed model is organized in accordance with the theoretical progression outlined in Table 1. Each lecture stage reflects a corresponding semiotic transition and APOS component, as presented in Table 2. The model is designed for undergraduate students studying higher mathematics in engineering and technical university programs. Mathematical exposition, guided questioning, dynamic visualization, prediction tasks, and systematic transitions between different forms of complex numbers are integrated within the lecture structure. Each stage of the lecture prepares the transition to the next form of representation, forming a didactic algorithm presented in Table 2.

Table 2. Didactic Structure of the Interactive Lecture Model

Stage	Didactic function	Main representation	Interactive element
Extension of number systems	Cognitive necessity	Symbolic	Guided questions
Imaginary unit	Introduction of new object	Symbolic	Conceptual discussion
Algebraic form	Formal representation	Algebraic	Guided examples
Complex plane	Geometric interpretation	Geometric	Prediction task
Trigonometric form	Coordinated representation	Algebraic-geometric	Visualization
Operations	Dynamic meaning	Trigonometric-geometric	Prediction and verification
Consolidation	Conceptual connection	Multiple	Representational comparison
Exponential form	Generalization	Trigonometric-exponential	Visualization and comparison

A progressive trajectory is followed throughout the lecture, driven by cognitive necessity toward geometric and trigonometric interpretation. Interaction is embedded directly into each stage, with the nature of engagement shifting from guided questioning and conceptual discussion in the opening stages toward prediction tasks, dynamic visualization, and comparison of equivalent forms as the lecture advances.

Epistemic motivation: extension of number systems

The lecture opens with equations whose solutions require gradual extension of number systems: require gradual extension of number systems:

$$x + 5 = 3, \quad 2x = 1, \quad x^2 = 2.$$

The successive introduction of the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ is examined through guided discussion, focusing on why each extension becomes necessary when equations cannot be solved within previously established number sets. The discussion proceeds to the equation $x^2 + 1 = 0$, through which the necessity of introducing a new number system is established. The imaginary unit is introduced through the relation

$$i^2 = -1 \quad (1)$$

At this stage, conceptual motivation takes precedence over formal definition. Complex numbers are presented as a natural continuation of the historical expansion of number systems, and this motivational foundation serves as the basis for the algebraic form introduced in the following stage.

Algebraic form of complex numbers

The motivational foundation established in the preceding stage leads naturally to the formal representation of complex numbers in algebraic form:

$$z = a + bi, \quad a, b \in \mathbb{R}. \quad (2)$$

Interpretation of the real and imaginary parts, $Re(z) = a$, $Im(z) = b$, is developed through guided discussion. Equality of complex numbers, purely imaginary numbers, and real numbers as special cases are examined. Operations are introduced gradually through guided examples and targeted questions about the structure of each operation. Addition and subtraction are interpreted componentwise, while multiplication is used to reinforce the meaning of relation (1). The periodicity of powers of the imaginary unit is identified through observation of the sequence $i, -1, -i, 1$. Division is introduced through conjugate numbers and transformation into standard algebraic form. Having established the algebraic form (2), the lecture proceeds to geometric interpretation in the complex plane.

Geometric representation in the complex plane

The transition from algebraic to geometric interpretation is motivated through the observation that a complex number contains two components and cannot therefore be represented adequately on a single number line. Complex numbers are represented simultaneously as points in the complex plane, as vectors originating from the coordinate origin, and as ordered pairs corresponding to real and imaginary components.

The number $z = 2 + 2i$, for example, is represented as the point $(2, 2)$, demonstrating the correspondence between algebraic and geometric representations shown in Figure 1.

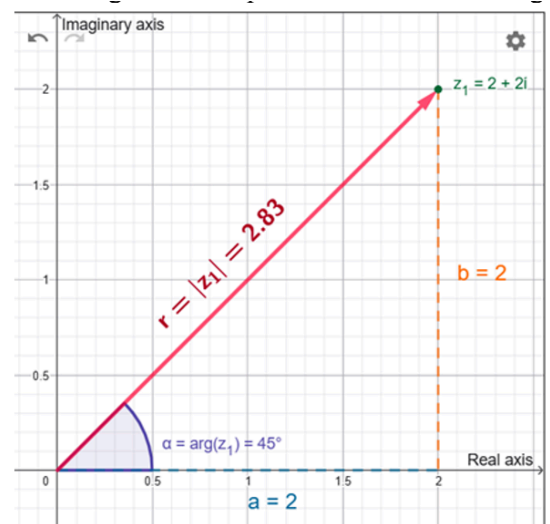


Fig. 1. Dynamic representation of a complex number z_1 , its modulus $|z_1|$, and argument $\arg(z_1)$ in the complex plane using GeoGebra

GeoGebra visualization is employed to connect symbolic and geometric forms dynamically. Students observe how changes in algebraic coordinates affect the position of the corresponding point in the complex plane. The modulus is introduced geometrically as the length of the radius vector:

$$|z| = \sqrt{a^2 + b^2}. \quad (3)$$

The argument is interpreted as the angle formed with the positive direction of the real axis. Interactive tasks at this stage include identifying quadrants, determining the modulus using relation (3),

distinguishing purely imaginary numbers, and interpreting points lying on coordinate axes.

Example interactive task

Students are asked to locate the following complex numbers in the complex plane: $z_1 = 3 + i$, $z_2 = -2 + 4i$, $z_3 = -3 - 2i$, $z_4 = 5i$. Prior to using GeoGebra, predictions are made regarding the quadrant of each number, which number is purely imaginary, which numbers lie on coordinate axes, and which has the greatest modulus. The numbers are then visualized dynamically and the results are discussed collectively, with attention to both the accuracy of predictions and the connections between algebraic and geometric forms.

This geometric foundation prepares the transition to the trigonometric form, in which modulus and argument serve as the primary parameters of representation.

Trigonometric form The trigonometric form is derived through geometric interpretation of the coordinates in the complex plane. Using $\cos \alpha = \frac{a}{|z|}$ and $\sin \alpha = \frac{b}{|z|}$, the coordinates are expressed as $a = |z| \cos \alpha$ and $b = |z| \sin \alpha$. Substitution into the algebraic form yields:

$$z = |z|(\cos \alpha + i \sin \alpha). \quad (4)$$

The emergence of the trigonometric form from geometric interpretation – rather than from isolated symbolic manipulation – is emphasized throughout this stage. GeoGebra supports simultaneous observation of the point in the complex plane, the radius vector, the modulus, the argument, and the trigonometric decomposition of coordinates, as illustrated in Figure 2.

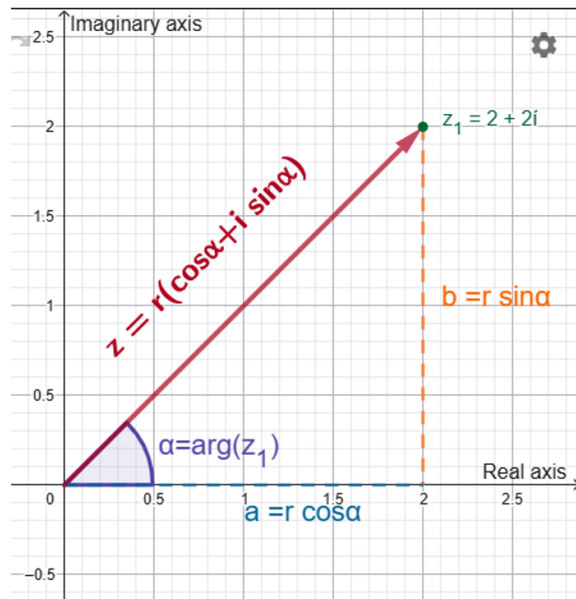


Fig. 2. Visualization of the trigonometric form of a complex number $z = r(\cos \alpha + i \sin \alpha)$ using GeoGebra

Trigonometric representation (4) prepares the geometric interpretation of operations, in which modulus and argument become the primary parameters through which multiplication and division are understood.

Operations as dynamic transformations

Operations with complex numbers in trigonometric form are interpreted geometrically. For $z_1 = r(\cos \alpha + i \sin \alpha)$ and $z_2 = p(\cos \beta + i \sin \beta)$ the product is derived as:

$$z_1 z_2 = rp(\cos(\alpha + \beta) + i \sin(\alpha + \beta)) \quad (5)$$

The geometric meaning of result (5) is emphasized: moduli are multiplied and arguments are added. Dynamic observation of multiplication as simultaneous rotation and scaling of vectors in the complex plane is supported through GeoGebra. The geometric meaning is reinforced through the interpretation of $i = 1(\cos 90^\circ + i \sin 90^\circ)$ leading to: $i^2 = -1$.

Division is interpreted analogously through subtraction of arguments:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\alpha - \beta) + i \sin(\alpha - \beta)) \quad (6)$$

Example interactive task

The expression $\frac{1+i}{1-i}$ is investigated. Before any calculation, students are asked to predict geometrically how division will affect the modulus and argument of the result – specifically, what occurs when two complex numbers with equal moduli are divided, and how their arguments combine. The trigonometric forms are then constructed independently: $1+i = \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$ and $1-i = \sqrt{2}(\cos(-45^\circ) + i \sin(-45^\circ))$.

The quotient is interpreted geometrically through subtraction of arguments in accordance with relation (6), yielding $\frac{1+i}{1-i} = i$.

The result is verified algebraically, allowing comparison of symbolic and geometric reasoning. The dynamic visualization shown in Figure 3 illustrates multiplication as simultaneous rotation and scaling in the complex plane.

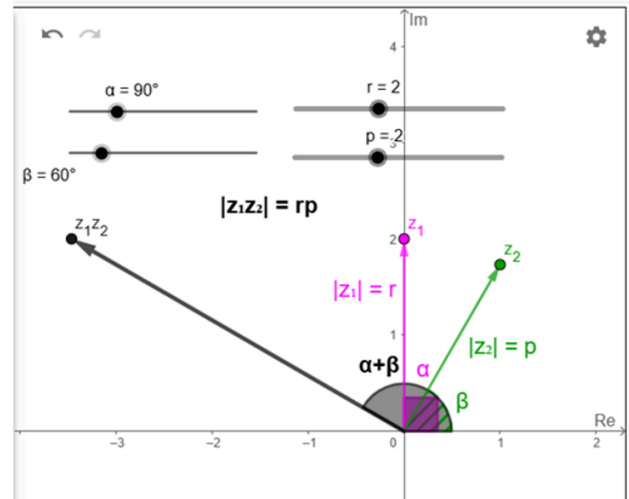


Fig. 3. Dynamic interpretation of multiplication of complex numbers as rotation and scaling using GeoGebra

The geometric interpretation established at this stage serves as the foundation for the analysis of powers and roots in the following section.

Powers, roots, and exponential form of complex numbers

Repeated multiplication in trigonometric form is examined to identify regularities leading to De Moivre's formula. Rather than presenting the formula directly, students are encouraged to analyze successive powers geometrically and to predict relationships between modulus and argument during exponentiation. Through this inductive process, the following result is reached:

$$z^n = r^n(\cos(n\alpha) + i \sin(n\alpha)). \quad (7)$$

Particular attention is devoted to the observation that relation (7) implies complex numbers with arguments differing by multiples of 360° represent the same point in the complex plane. This idea is used to introduce roots through geometric interpretation. Equations of the form

$$w^n = z \quad (8)$$

are investigated, and students are asked to predict the number and arrangement of solutions before the root formula is formally derived. GeoGebra supports interpretation of roots as equally spaced points on a circle in the complex plane, and dynamic manipulation of the argument allows observation of how different roots produce the same complex number after exponentiation (Figure 4).

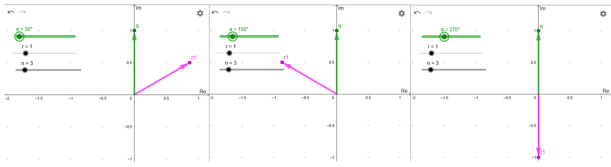


Fig. 4. Visualization of roots of complex numbers in the complex plane

The trigonometric form leads naturally to the exponential representation of complex numbers. Through Euler's formula:

$$e^{i\alpha} = \cos \alpha + i \sin \alpha, \quad (9)$$

the trigonometric form $z = r(\cos \alpha + i \sin \alpha)$ is rewritten as:

$$z = re^{i\alpha}. \quad (10)$$

The retention of geometric meaning by modulus and argument in representation (10) is observed and discussed. GeoGebra visualization supports comparison of the trigonometric and exponential forms, reinforcing the connection between algebraic structure and geometric interpretation. The exponential form is presented not as an isolated symbolic expression, but as a generalization unifying the preceding representations within a single compact notation.

Integration of GeoGebra as a mediating tool

Within the proposed model, GeoGebra functions as a mediating tool supporting coordination between algebraic, geometric, trigonometric, and exponential forms of complex numbers. Beyond its role as a visualization environment, the software supports dynamic transitions between semiotic representations, conceptual interpretation, and guided interaction during the lecture. Simultaneous display of algebraic and geometric forms, dynamic manipulation of modulus and argument, interpretation of geometric transformations, prediction and verification tasks, and visualization of powers and roots are all supported through GeoGebra. The software thereby mediates transitions between multiple semiotic registers and supports the gradual construction of conceptual connections, enabling comparison of symbolic, geometric, trigonometric, and exponential interpretations within a single dynamic environment.

Consolidation: conceptual connection between representations

The concluding stage of the lecture is devoted to explicit consolidation of connections between algebraic, geometric, trigonometric, and exponential representations of complex numbers. Students are invited to reflect on the relationships between representations encountered during the lecture and to discuss how each form contributes to a unified understanding of complex numbers as mathematical objects. Guided questions focus on coordinating symbolic, geometric, trigonometric, and exponential reasoning within a single conceptual framework. Interactive activities at this stage emphasize comparison, interpretation, and explanation of equivalent representations – in contrast to the routine calculations that characterized earlier stages. GeoGebra visualizations from previous stages may be revisited to support representational synthesis during classroom discussion.

This stage corresponds to the Schema component within the APOS perspective, in which complex numbers are understood as mathematical objects coordinated across multiple representational forms. The progression from epistemic motivation through algebraic, geometric, trigonometric, and exponential representation toward explicit consolidation reflects the core didactic principle of the proposed model: conceptual understanding develops through coordinated transitions between representations, not through any single form in isolation. Formative assessment is integrated throughout the lecture through prediction tasks, conceptual questioning, representational comparison, and immediate verification using GeoGebra visualization.

5. RESULTS AND DISCUSSION

The proposed model integrates algebraic, geometric, trigonometric, and exponential representations within a coordinated interactive lecture structure. Dynamic visualization is combined with

guided questioning, prediction tasks, and systematic transitions between representations, with formative feedback provided through verification and discussion of equivalent forms rather than through separate assessment activities. This organization addresses difficulties reported in previous studies concerning fragmentation between symbolic and geometric reasoning [1–4] and extends existing GeoGebra-supported approaches by incorporating visualization into lecture interaction rather than using it solely for demonstration. The progressive structure of the model – from epistemic motivation through coordinated representational transitions toward explicit consolidation – reflects the theoretical progression outlined in Table 1 and is designed to support the gradual construction of complex numbers as coherent mathematical objects.

The contribution of the present study lies in the development of a structured lecture framework combining coordinated representations, dynamic visualization, and formative interaction within undergraduate mathematics instruction. The proposed framework has not been empirically validated at this stage, and no empirical conclusions regarding its effectiveness can therefore be drawn. The theoretical design, however, is grounded in established perspectives on concept construction and representational coordination, and the instructional sequence has been developed with attention to the specific difficulties associated with learning complex numbers identified in the literature [8, 12, 13, 14].

An interactive lecture based on the proposed model has been prepared and is planned for implementation with first-year students in engineering specialties at the Technical University of Gabrovo. The planned empirical study will investigate both conceptual understanding of complex numbers across their algebraic, geometric, trigonometric, and exponential forms, and the ability to coordinate transitions between representational registers. Pre-test and post-test designs will be employed to assess the effectiveness of the proposed lecture organization, and comparisons with traditional lecture formats are anticipated. The results of this empirical investigation will provide evidence regarding the pedagogical potential of coordinated representational instruction supported by dynamic visualization in undergraduate engineering mathematics education.

6. CONCLUSIONS

A structured interactive lecture model for teaching complex numbers in undergraduate mathematics education using GeoGebra has been proposed in the present study. Algebraic, geometric, trigonometric, and exponential representations are integrated within a coordinated instructional sequence supported by dynamic visualization, guided classroom interaction, and formative assessment embedded throughout the lecture process.

The instructional sequence is theoretically grounded in APOS theory and the theory of semiotic representations. The progressive organization – from cognitive necessity through procedural action toward a coordinated schema – reflects the gradual construction of complex numbers as mathematical objects through systematic transitions between symbolic, geometric, trigonometric, and exponential forms. Within this framework, GeoGebra functions as a mediating tool supporting transitions between representational forms, extending beyond its conventional role as a demonstration environment.

The developed framework is applicable in undergraduate mathematics courses for engineering and technical university students and is adaptable for other mathematical topics involving multiple representations and geometric interpretation. It also serves as a structural template for integrating theoretical frameworks into lecture design across other areas of undergraduate mathematics.

Empirical validation of the proposed model is planned through classroom implementation with engineering students at the Technical University of Gabrovo. Conceptual understanding of complex numbers across multiple representational forms and the ability to coordinate transitions between representational registers will both be investigated. Pre-test and post-test designs will be employed, and comparative studies involving traditional and interactive lecture formats will provide further evidence regarding the effectiveness of coordinated representational instruction supported by dynamic vi-

sualization in undergraduate engineering mathematics education.

REFERENCES

- [1] **Akveld M., Docherty P., Kinnear G. et al.** *Question Design for Classroom Voting in Undergraduate Mathematics: A Systematic Review of the Literature*, International Journal of Research in Undergraduate Mathematics Education, 2025.
- [2] **Breda A. M. D., Dos Santos Dos Santos J. M.** *Learning Complex Functions with GeoGebra*, Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales, Vol. 45, 2021, Article 1504.
- [3] **Breda A. M. D., Santos J. M. dos S.** *Complex Functions with GeoGebra*, Teaching Mathematics and its Applications: An International Journal of the IMA, Vol. 35, No. 2, 2016, pp. 102–110.
- [4] **Drijvers P., Sinclair N.** *The Role of Digital Technologies in Mathematics Education: Purposes and Perspectives*, ZDM Mathematics Education, Vol. 56, 2024, pp. 239–248.
- [5] **Dubinsky E., McDonald M. A.** *APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research*, in The Teaching and Learning of Mathematics at University Level, Holton D. (Ed.), Kluwer, 2001, pp. 275–282.
- [6] **Duval R.** *A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics*, Educational Studies in Mathematics, Vol. 61, 2006, pp. 103–131.
- [7] **Johnson E., Weber K., Fukawa-Connelly T. P. et al.** *Collaborating with Mathematicians to Use Active Learning in University Mathematics Courses: The Importance of Attending to Mathematicians' Obligations*, Educational Studies in Mathematics, Vol. 119, 2025, pp. 145–161.
- [8] **Mutambara L. H. N., Tsakeni M.** *Cognitive Obstacles in the Learning of Complex Number Concepts: A Case Study of In-Service Undergraduate Physics Student-Teachers in Zimbabwe*, Eurasia Journal of Mathematics, Science and Technology Education, Vol. 18, No. 10, 2022, Article em2158.
- [9] **Ní Shé C., Ní Fhloinn E., Mac an Bhaird C.** *Student Engagement with Technology-Enhanced Resources in Mathematics in Higher Education: A Review*, Mathematics, Vol. 11, No. 3, 2023, Article 787.
- [10] **Papageorgiou E., Wong J., Liu Q. et al.** *A Systematic Review on Student Engagement in Undergraduate Mathematics: Conceptualization, Measurement, and Learning Outcomes*, Educational Psychology Review, Vol. 37, 2025, Article 66.
- [11] **Petropoulou G., Jaworski B., Potari D. et al.** *Undergraduate Mathematics Teaching in First Year Lectures: Can It Be Responsive to Student Learning Needs?*, International Journal of Research in Undergraduate Mathematics Education, Vol. 6, 2020, pp. 347–374.
- [12] **Seloane M., Ramaila S., Ndlovu M.** *Developing Undergraduate Engineering Mathematics Students' Conceptual and Procedural Knowledge of Complex Numbers Using GeoGebra*, Pythagoras, Vol. 44, No. 1, 2023, Article 763.
- [13] **Seloane M., Ramaila S., Ndlovu M.** *Harnessing GeoGebra as a Modelling Tool to Mitigate Undergraduate Engineering Mathematics Students' Misconceptions and Errors Associated with Complex Numbers*, Pythagoras, Vol. 46, No. 1, 2025, Article 825.
- [14] **Smith E. M., Zwolak J. P., Manogue C. A.** *Isolating Approaches: How Middle-Division Physics Students Coordinate Forms and Representations in Complex Algebra*, Physical Review Physics Education Research, Vol. 15, No. 1, 2019, Article 010138.
- [15] **Zhang Y., Wang P., Jia W., Zhang A., Chen G.** *Dynamic Visualization by GeoGebra for Mathematics Learning: A Meta-Analysis of 20 Years of Research*, Journal of Research on Technology in Education, Vol. 57, No. 2, 2025, pp. 437–458.